New Items and Inference Rules for 3d Grammars

Theory Group

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1 Item Semantics

Given a grammar G and a string S, items are of the form $[t, \eta, X, i, j, k, l]$, where:

- t is the index of a local tree in G;
- η is one of ε, 0, 1, or 2;
- X is a symbol from G;
- i, j, k, l are indices in the range $[0, |S|]$ in increasing order or $\emptyset$.

The values of η denote local tree positions, where ε is interpreted as the root of the local tree, 0 denotes the root of its yield, 1 refers to the left or only child, and 2 indicates the right child.

The presence of the item $[t, \eta, X, i, j, k, l]$ indicates that, under Grammar G, the symbol X licenses (dominates in the 2nd or 3rd dimension) the substring of S indexed by the range $[i, j)$ (the substring "to the left of the spine") and the substring of S indexed by the range $[k, l)$ (the substring "to the right of the spine"), provided that X occurs at position η of local tree t.

If i and j are both $\emptyset$, then the first substring is taken to be any empty substring of S. If k and l are both $\emptyset$, then the second substring is taken to be any empty substring of S. If i and l are indices but j and k are both $\emptyset$, then X is instead taken to license a single substring of S indexed by the range $[i, l)$. These possibilities notwithstanding, all four of i, j, k, l must be indices.

2 Quick Overview

We have two axioms and 21 inference rules, which are really all variations on 3 basic inference rules. The axioms denote items for the trivial trees permitted by the grammar. Items are constructed for the other local trees using the inference rules.

First, inference rules 1-5 are applied to construct items for the leaves (Y and Z) of the local tree. The antecedents for these rules are the items for the roots of other local trees or for the axioms (which are also items for the roots of other local trees).

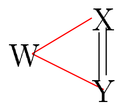
Second, inference rules 6-11 are applied to construct items for the root of the yield (X). The antecedents for these rules are the items for the leaves (Y and Z) of the same local tree. Because rule 6 covers the case where X has no children in the 2nd dimension, it therefore has no antecedents and is technically an axiom, but it is written as rule 6 to emphasize the fact that it is really another variation of rules 7-11.

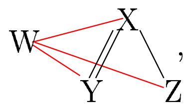
Finally, inference rules 12-21 are applied to construct items for the root of the local tree (W). The antecedents for these rules are the items for the root of the yield (X) and for the root of the local tree that is adjoined at X (which again may be a trivial tree and hence axiomatic). Rule 12 is a special case where X has no children in the 2nd dimension; a special case is needed here because the adjoined tree is permitted to have no gap in this case, a condition that must be checked for and prevented when X does have children. Also, note that one of the antecedents of rule 12 is simply the result of the axiomatic rule 6, and that rules 6 and 12 are licensed under the same side-conditions. As a result, rules 6 and 12 could be combined into a single rule with no effect on the soundness or completeness of the system, but they have been written in this manner for consistency with the 3-step mechanism used to derive the items for the other types of local trees.

3 Axioms

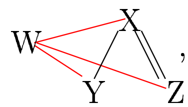
1. $[t, \varepsilon, x, i, \emptyset, \emptyset, i+1]$: $t = x\cdot, S[i] = x$
2. $[t, \varepsilon, X, \emptyset, \emptyset, \emptyset, \emptyset]$: $t = X\cdot, X \notin \Sigma$

4 Inference Rules

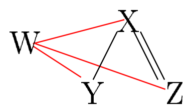
1. $\dfrac{[t_2,\varepsilon,Y,i,j,k,l]}{[t_1,1,Y,i,j,k,l]}$: $t_1 =$ 

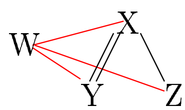
2. $\dfrac{[t_2,\varepsilon,Y,i,j,k,l]}{[t_1,1,Y,i,j,k,l]}$: $t_1 =$ 

$, j = \emptyset \rightarrow i = \emptyset$

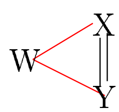
3. $\dfrac{[t_2,\varepsilon,Z,i,j,k,l]}{[t_1,2,Z,i,j,k,l]}$: $t_1 =$ 

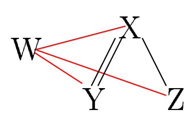
$, j = \emptyset \rightarrow i = \emptyset$

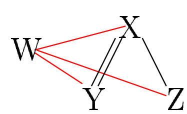
4. $\dfrac{[t_2,\varepsilon,Y,i,\emptyset,\emptyset,l]}{[t_1,1,Y,i,\emptyset,\emptyset,l]}$: $t_1 =$ 

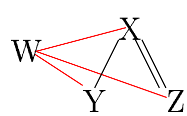
5. $\dfrac{[t_2,\varepsilon,Z,i,\emptyset,\emptyset,l]}{[t_1,2,Z,i,\emptyset,\emptyset,l]}$: $t_1 =$ 

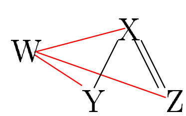
6. $\dfrac{}{[t,0,X,\emptyset,\emptyset,\emptyset,\emptyset]}$: $t =$ 

7. $\dfrac{[t,1,Y,i,j,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

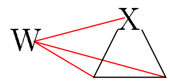
8. $\dfrac{[t,1,Y,i,j,k,m][t,2,Z,m,\emptyset,\emptyset,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

9. $\dfrac{[t,1,Y,i,j,\emptyset,\emptyset][t,2,Z,k,\emptyset,\emptyset,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

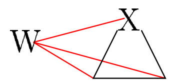
10. $\dfrac{[t,1,Y,i,\emptyset,\emptyset,n][t,2,Z,n,j,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

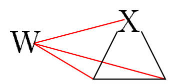
11. $\dfrac{[t,1,Y,i,\emptyset,\emptyset,j][t,2,Z,\emptyset,\emptyset,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

12. $\dfrac{[t_1,0,X,\emptyset,\emptyset,\emptyset,\emptyset][t_2,\varepsilon,X,i,j,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

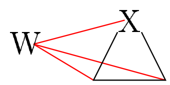
13. $\frac{[t_1,0,X,n,j,k,m][t_2,\varepsilon,X,i,n,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

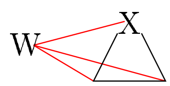
, $n = \emptyset \to i = \emptyset$

14. $\frac{[t_1,0,X,i,j,k,m][t_2,\varepsilon,X,\emptyset,\emptyset,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

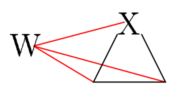
15. $\frac{[t_1,0,X,\emptyset,\emptyset,k,m][t_2,\varepsilon,X,i,j,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

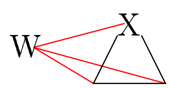
, $j = \emptyset \to i = \emptyset$

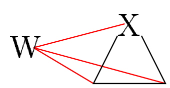
16. $\frac{[t_1,0,X,n,j,k,l][t_2,\varepsilon,X,i,n,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

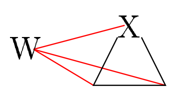
17. $\frac{[t_1,0,X,n,j,\emptyset,\emptyset][t_2,\varepsilon,X,i,n,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

, $n = \emptyset \to i = \emptyset$

18. $\frac{[t_1,0,X,i,j,\emptyset,\emptyset][t_2,\varepsilon,X,\emptyset,\emptyset,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

19. $\frac{[t_1,0,X,\emptyset,\emptyset,k,l][t_2,\varepsilon,X,i,j,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

20. $\frac{[t_1,0,X,i,j,k,l][t_2,\varepsilon,X,\emptyset,\emptyset,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

21. $\frac{[t_1,0,X,\emptyset,\emptyset,\emptyset,\emptyset][t_2,\varepsilon,X,i,j,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}: t_1 =$ 

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