New Items and Inference Rules for 3d Grammars

Theory Group

June 21, 2004

1 Item Semantics

Given a grammar G and a string S, items are of the form $[t, \eta, X, i, j, k, l]$, where:

- t is the index of a local tree in G;
- η is one of ε, 0, 1, or 2;
- X is a symbol from G;
- i, j, k, l are indices in the range $[0, |S|]$ in increasing order or $\emptyset$.

The values of η denote local tree positions, where ε is interpreted as the root of the local tree, 0 denotes the root of its yield, 1 refers to the left or only child, and 2 indicates the right child.

The presence of the item $[t, \eta, X, i, j, k, l]$ indicates that, under Grammar G, the symbol X licenses (dominates in the 2nd or 3rd dimension) the substring of S indexed by the range $[i, j)$ (the substring "to the left of the spine") and the substring of S indexed by the range $[k, l)$ (the substring "to the right of the spine"), provided that X occurs at position η of local tree t. In the case of items that occur at position η, 1, or 2, the licensing occurs in the third dimension, while at position 0, the licensing occurs in the second dimension.

If i and j are both $\emptyset$, then the first substring is taken to be any empty substring of S. If k and l are both $\emptyset$, then the second substring is taken to be any empty substring of S. If i and l are indices but j and k are both $\emptyset$, then X is instead taken to license a single substring of S indexed by the range $[i, l)$. These possibilities notwithstanding, all four of i, j, k, l must be indices.

2 Quick Overview

We have two axioms and 23 inference rules, which are really all variations on 3 basic inference rules. The axioms denote items for the trivial trees permitted by the grammar. Items are constructed for the other local trees using the inference rules.

First, inference rules 1-5 are applied to construct items for the leaves (Y and Z) of the local tree. The antecedents for these rules are the items for the roots of other local trees or for the axioms (which are also items for the roots of other local trees).

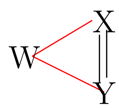
Second, inference rules 6-13 are applied to construct items for the root of the yield (X). The antecedents for these rules are the items for the leaves (Y and Z) of the same local tree. Because rule 6 covers the case where X has no children in the 2nd dimension, it therefore has no antecedents and is technically an axiom, but it is written as rule 6 to emphasize the fact that it is really another variation of rules 7-13.

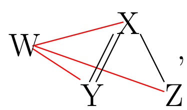
Finally, inference rules 14-23 are applied to construct items for the root of the local tree (W). The antecedents for these rules are the items for the root of the yield (X) and for the root of the local tree that is adjoined at X (which again may be a trivial tree and hence axiomatic). Rule 14 is a special case where X has no children in the 2nd dimension; a special case is needed here because the adjoined tree is permitted to have no gap in this case, a condition that must be checked for and prevented when X does have children. Also, note that one of the antecedents of rule 14 is simply the result of the axiomatic rule 6, and that rules 6 and 14 are licensed under the same side-conditions. As a result, rules 6 and 14 could be combined into a single rule with no effect on the soundness or completeness of the system, but they have been written in this manner for consistency with the 3-step mechanism used to derive the items for the other types of local trees.

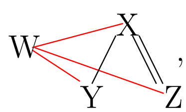
3 Axioms

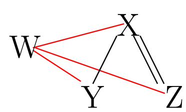
1. $[t, \varepsilon, x, i, \emptyset, \emptyset, i+1]$: $t = x\cdot, S[i] = x$
2. $[t, \varepsilon, X, \emptyset, \emptyset, \emptyset, \emptyset]$: $t = X\cdot, X \notin \Sigma$

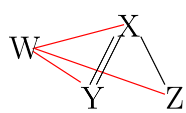
4 Inference Rules

1. $\frac{[t_2,\varepsilon,Y,i,j,k,l]}{[t_1,1,Y,i,j,k,l]}$: $t_1 =$ 

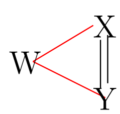
2. $\frac{[t_2,\varepsilon,Y,i,j,k,l]}{[t_1,1,Y,i,j,k,l]}$: $t_1 =$  $, j = \emptyset \rightarrow i = \emptyset$

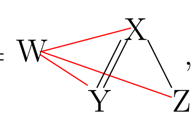
3. $\frac{[t_2,\varepsilon,Z,i,j,k,l]}{[t_1,2,Z,i,j,k,l]}$: $t_1 =$  $, j = \emptyset \rightarrow i = \emptyset$

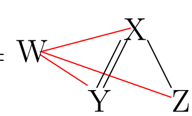
4. $\frac{[t_2,\varepsilon,Y,i,\emptyset,\emptyset,l]}{[t_1,1,Y,i,\emptyset,\emptyset,l]}$: $t_1 =$ 

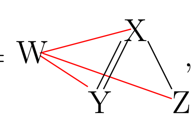
5. $\frac{[t_2,\varepsilon,Z,i,\emptyset,\emptyset,l]}{[t_1,2,Z,i,\emptyset,\emptyset,l]}$: $t_1 =$ 

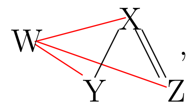
6. $\frac{}{[t,0,X,\emptyset,\emptyset,\emptyset,\emptyset]}$: $t =$ 

7. $\frac{[t,1,Y,i,j,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

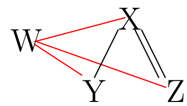
8. $\frac{[t,1,Y,i,j,k,m][t,2,Z,m,\emptyset,\emptyset,l]}{[t,0,X,i,j,k,l]}$: $t =$  $, j = \emptyset \rightarrow i = \emptyset$

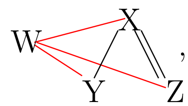
9. $\frac{[t,1,Y,i,j,\emptyset,\emptyset][t,2,Z,k,\emptyset,\emptyset,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

10. $\frac{[t,1,Y,i,j,k,l][t,2,Z,\emptyset,\emptyset,\emptyset,\emptyset]}{[t,0,X,i,j,k,l]}$: $t =$  $, j = \emptyset \rightarrow i = \emptyset$

11. $\dfrac{[t,1,Y,i,\emptyset,\emptyset,n][t,2,Z,n,j,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

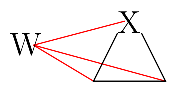
, $j = \emptyset \rightarrow n = \emptyset$

12. $\dfrac{[t,1,Y,i,\emptyset,\emptyset,j][t,2,Z,\emptyset,\emptyset,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

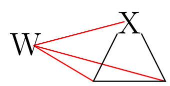
13. $\dfrac{[t,1,Y,\emptyset,\emptyset,\emptyset,\emptyset][t,2,Z,i,j,k,l]}{[t,0,X,i,j,k,l]}$: $t =$ 

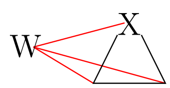
, $j = \emptyset \rightarrow i = \emptyset$

14. $\dfrac{[t_1,0,X,\emptyset,\emptyset,\emptyset,\emptyset][t_2,\varepsilon,X,i,j,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

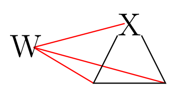
15. $\dfrac{[t_1,0,X,n,j,k,m][t_2,\varepsilon,X,i,n,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

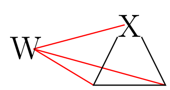
, $n = \emptyset \rightarrow i = \emptyset$

16. $\dfrac{[t_1,0,X,i,j,k,m][t_2,\varepsilon,X,\emptyset,\emptyset,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

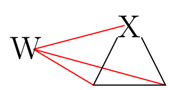
17. $\dfrac{[t_1,0,X,\emptyset,\emptyset,k,m][t_2,\varepsilon,X,i,j,m,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

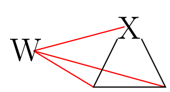
, $j = \emptyset \rightarrow i = \emptyset$

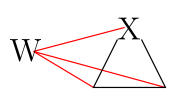
18. $\dfrac{[t_1,0,X,n,j,k,l][t_2,\varepsilon,X,i,n,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

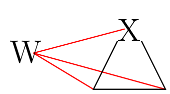
19. $\dfrac{[t_1,0,X,n,j,\emptyset,\emptyset][t_2,\varepsilon,X,i,n,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

, $n = \emptyset \rightarrow i = \emptyset$

20. $\dfrac{[t_1,0,X,i,j,\emptyset,\emptyset][t_2,\varepsilon,X,\emptyset,\emptyset,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

21. $\dfrac{[t_1,0,X,\emptyset,\emptyset,k,l][t_2,\varepsilon,X,i,j,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

22. $\dfrac{[t_1,0,X,i,j,k,l][t_2,\varepsilon,X,\emptyset,\emptyset,\emptyset,\emptyset]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

23. $\dfrac{[t_1,0,X,\emptyset,\emptyset,\emptyset,\emptyset][t_2,\varepsilon,X,i,j,k,l]}{[t_1,\varepsilon,W,i,j,k,l]}$: $t_1 =$ 

, $j = \emptyset \rightarrow i = \emptyset$

5 Soundness Proof

5.1 Soundness of Axioms

Axiom 1, $[t, \varepsilon, x, i, \emptyset, \emptyset, i+1]$, asserts that x licenses the single substring of S indexed by $[i, i+1)$ when it occurs at position ε of the local tree $x\cdot$ and when $S[i] = x$. This is true and therefore sound.

Axiom 2, $[t, \varepsilon, X, \emptyset, \emptyset, \emptyset, \emptyset]$, asserts that X licenses an empty substring of S when it occurs at position ε of the local tree $X\cdot$, provided that X is a non-terminal. Since the local tree $X\cdot$ has no successors in any dimension, this is true and therefore sound.

5.2 Soundness of Inference Rules 1-5

Rule 1 permits us to infer the yield of the two-dimensional child of a local tree in the case that it is the only such child. In this case, the inference is simple: any local tree rooted in Y may potentially be adjoined here, either with a spinal gap or without, and so there are no necessary side conditions on the inference. Rule 1 preserves theoremhood.

Rules 2 and 3 cover the case where there are two such two-dimensional children, and the node of interest is located on the spine. The node is not permitted to have a spinal gap in this case (provable by induction on the structure of the 3d trees, because the gap must be filled by an initial tree with fewer than 2 two-dimensional children, and that initial tree must occur higher in the tree than any adjoined tree), and the side condition $j = \emptyset \rightarrow i = \emptyset$ ensures that this prerequisite is met. In any other case, the indices can be imported directly from the root of the adjoined tree, and so Rules 2 and 3 are sound.

Rules 4 and 5 cover the opposite case, where the node of interest is located off the spine. This node will be the root of its own subspine, and so it is required either not to have a spinal gap or to have an empty yield. The rules cover exactly these cases, so Rules 4 and 5 are sound.

5.3 Soundness of Inference Rules 6-13

Rules 6-11 are used to infer the 2nd-dimensional yields of the 2nd-dimensional roots of the local trees. Rule 6 permits the derivation of an empty 2nd-dimensional yield for the 2nd-dimensional root in the case that there are no

2nd-dimensional children, which is a sound inference, and it permits no other inferences.

Rule 7 propagates the yield from the child to the 2nd-dimensional root in the case that there is only one 2nd-dimensional child. Again, as with Rule 1, all cases are valid and this rule simply propagates the data unchanged. It is therefore sound.

Rules 8-13 merge the yield from each child to the 2nd-dimensional root when there are two such children. Again, each of the rules requires the spinal child to have a spinal gap and the other child to have no gap. In each case, the side of the spine opposite the side of the non-spinal child simply propagates upward unchanged. For the other side, the ranges are merged if they both exist (Rules 8 and 11), or propagated if only one exists (Rules 9, 10, 12, 13) or if neither exists (any of Rules 8-11). These are all sound inferences, and no other inferences are permitted.

5.4 Soundness of Inference Rules 14-23

Rule 14 propagates the yield from the 3rd-dimensional successor to the root of the tree in the case that there are no two-dimensional children. Such a rule is essentially a renaming rule, and any yield is simply propagated unchanged. Rule 14 is sound.

Rules 15-23 are permitted when the local tree does have two-dimensional children. In this case, the tree being adjoined is required to have a gap (because the two-dimensional yield will go inside that gap), and the side conditions enforce this. The other antecedent, the two-dimensional yield of the node in position 0, may have a gap if it has one two-dimensional child but may not if there are two. The only rules that may potentially form this antecedent are rules 6 through 13. The only one of these that may form an item without a gap is rule 7. If rule 7 is licensed to operate on tree t, then tree t must have exactly one child, and therefore it will be okay to use such an item for tree t with rules 15-23. Thus we can be sure that the item for the two-dimensional yield is properly formed. The only rules that will take a non-gapped antecedent are rules 15, 16, 18, and 22, and inspection shows that each of these rules produces a sound inference when applied to such an item.

In each case remaining, each rule merges the ranges for the sides that exist in both antecedent and propagates the ranges that are absent from one or both. Thus, the remaining cases are also sound.

The axioms and all the inference rules are sound, so the system as a whole is sound.

6 Completeness Proof

Suppose a tree exists using grammar G that licenses S. To construct the item giving the yield of the root of the tree, first recursively construct the items giving the yields of the local trees adjoined to it. The local tree will now be one of the following cases:

1. The tree is a trivial tree $X\cdot$, containing only a single symbol X and having no adjoined trees.

2. The tree is a renaming tree W——X.

3. The tree contains one second-dimensional child.

4. The tree contains two second-dimensional children.

In case 1, if the tree is valid then it must be described by an item generated from either axiom 1, if the root is a terminal, or axiom 2, if the root is a non-terminal.

In case 2, an item can be generated for the (empty) two-dimensional yield of X using rule 6. The item for the root is then generated by using rule 14 and the item for the local tree adjoined to X to propagate the three-dimensional yield of X up as the yield of W.

In case 3, an item for the yield of the two-dimensional child (Y) can be generated using rule 1 and the item for the root of the tree adjoined at Y. Using rule 7, this item is propagated upward to become the item for the two-dimensional yield of the two-dimensional root (X). Finally, using one of rules 15-23 and the item for the tree adjoined at X, the two-dimensional yield is merged with the three-dimensional yield of X to obtain the item for the yield of the root W.

In this case, the item for X may index two substrings, one substring to the left of the spine, one substring to the right of the spine, one gapless substring, or no substring. The item for the adjoined tree may index two substrings, one substring to the left of the spine, one substring to the right of the spine, or no substring. Inspection reveals that, for each possible combination, at least one of rules 15-23 suffices.

In case 4, items for the two-dimensional children are generated using rules 2-5, much like the use of rule 1 in the previous case. Because one child is on the spine, and the other is not, one item will use either rule 2 or 3 and the other will use either rule 4 or 5. This means that one item will have a gap, and the other will not.

Next, using one of rules 8-13, a two-dimensional yield for X is again produced. If the spine is on the left, then rules 8-10 are used; if the spine is on the right, then the corresponding rules 11-13 are used. The child on the spine could license two substrings, one substring to the left, one substring to the right, or no substring. The child off the spine could license one gapped substring, or no substring. Again, inspection reveals that, for each possible combination, at least one of rules 8-13 suffices. In each case, the item produced contains a spinal gap.

Finally, one of rules 15-23 is applied along with the item for the tree adjoined at X to obtain the item for the yield of the root W. The possibilities here are the same as those in this stage of case 3, except that here the item for X must have a gap. Thus, rules 15-23 still suffice for all possible combinations.

The entire tree is constructed recursively upward using this method, and the set of inference rules is therefore complete.