Higher-Dimensional Rendering

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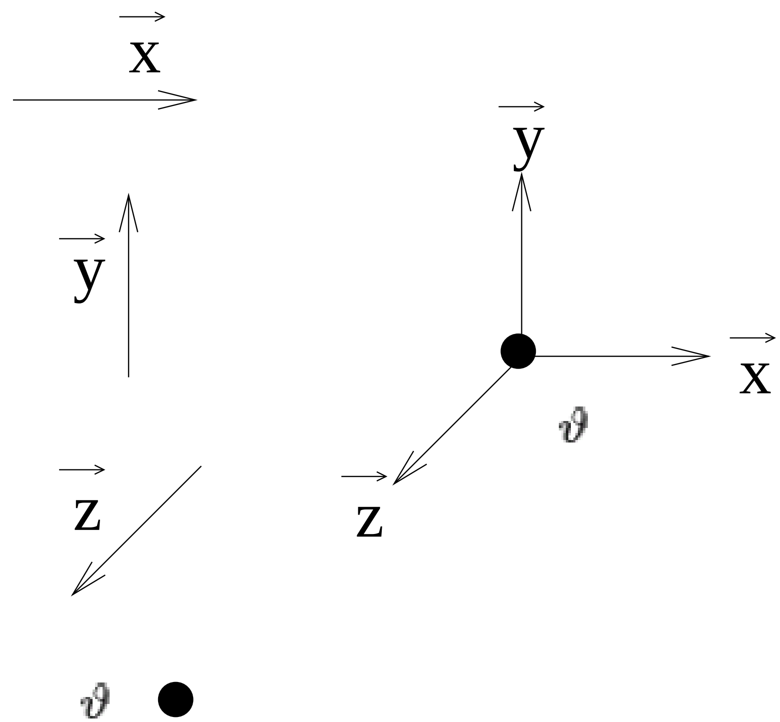
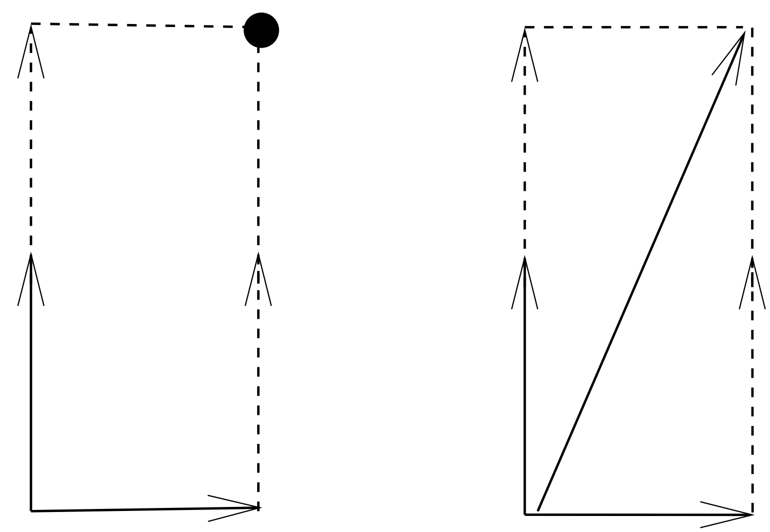
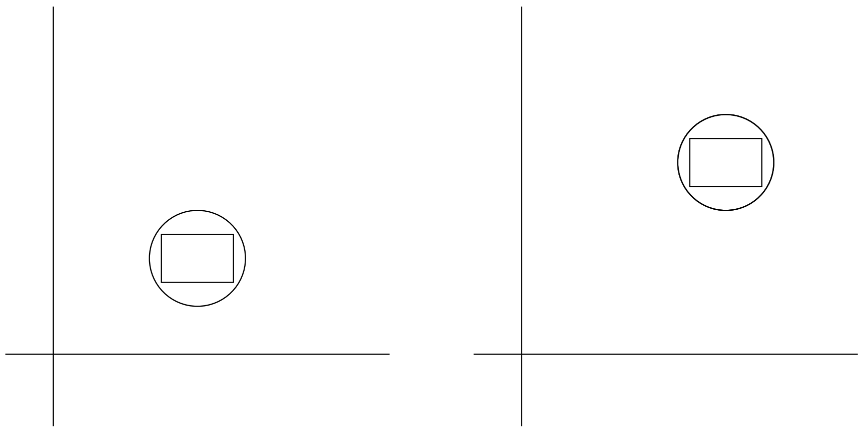
Coordinate Frame





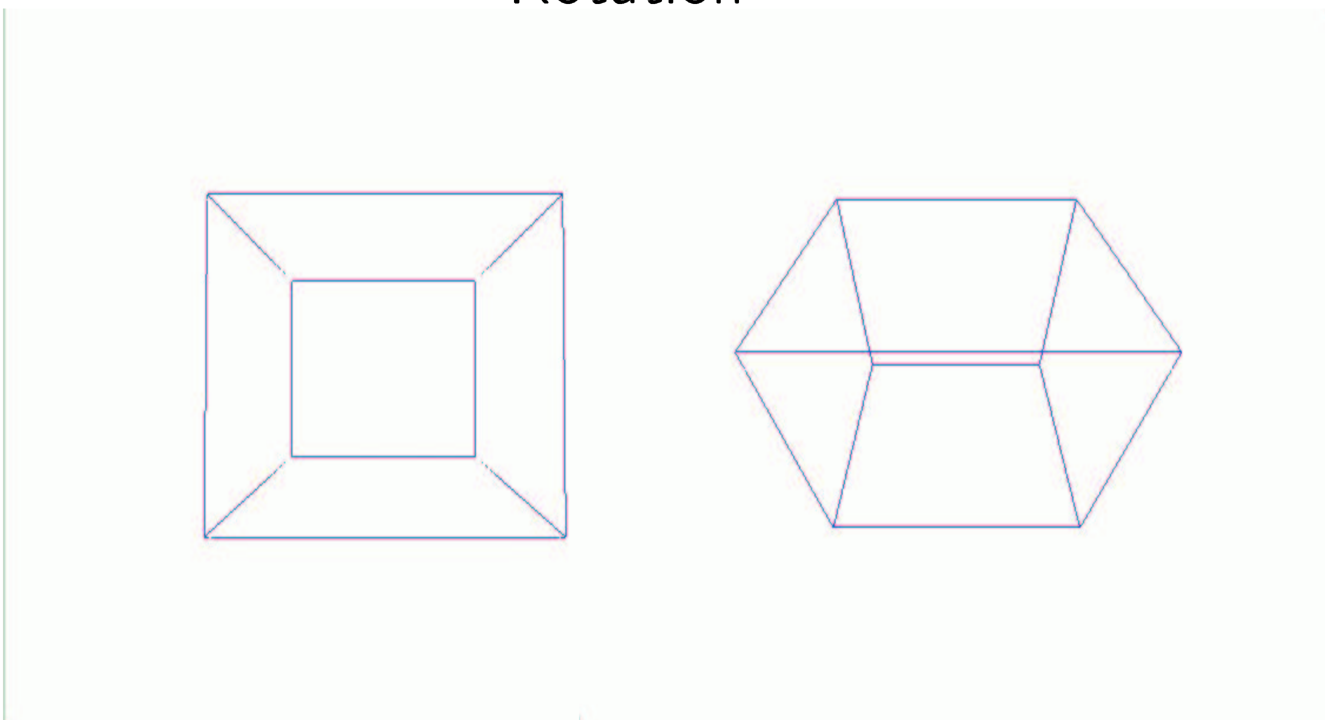
<1, 2, 1> <1, 2, 0>

Translation



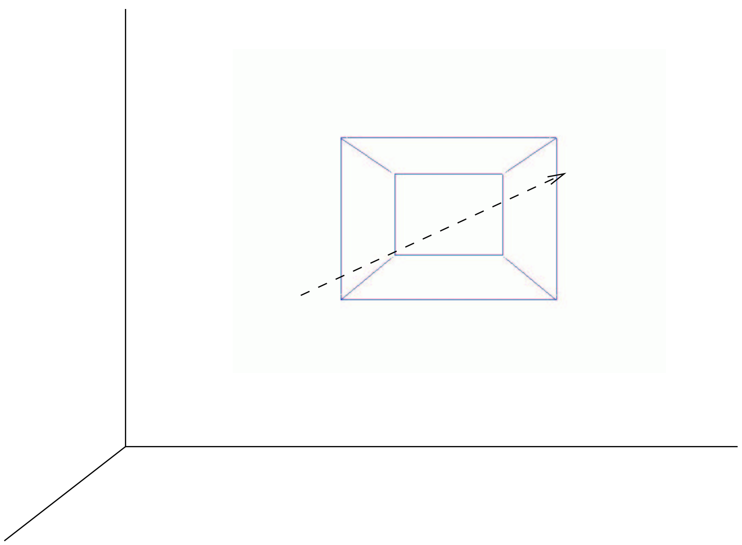
$$\begin{pmatrix} 1 & 0 & 0 & x_d \\ 0 & 1 & 0 & y_d \\ 0 & 0 & 1 & z_d \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_d \\ y + y_d \\ z + z_d \\ 1 \end{pmatrix}$$

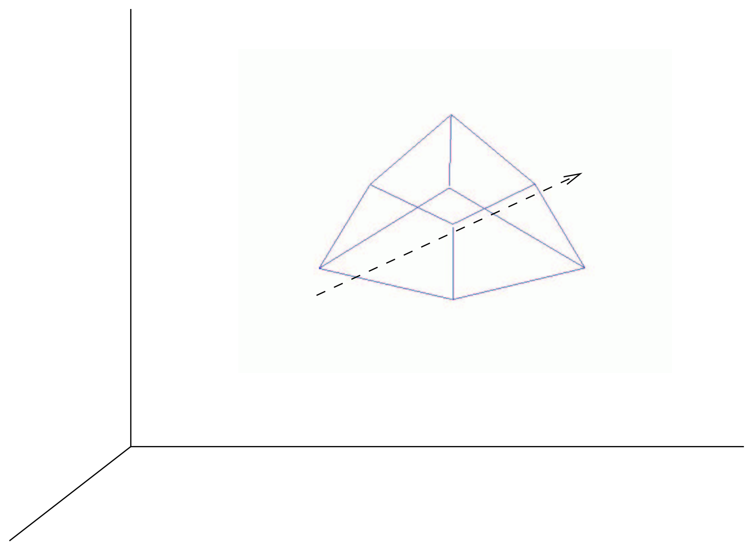
Rotation



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{pmatrix}$$

Rotation Around Arbitrary Vectors

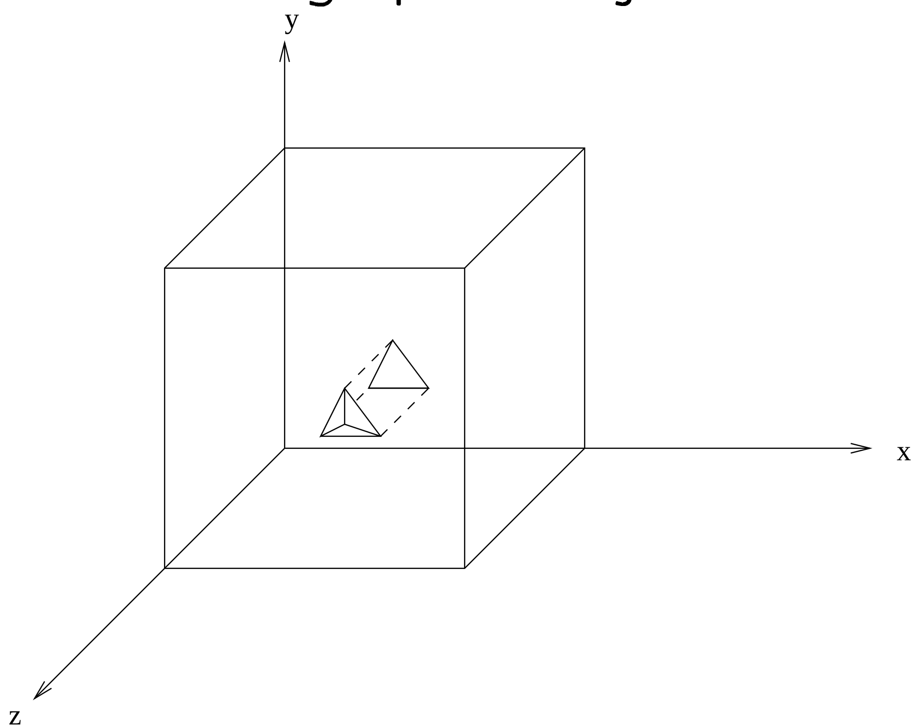




$$\vec{x} \times \vec{y} = \vec{n}$$

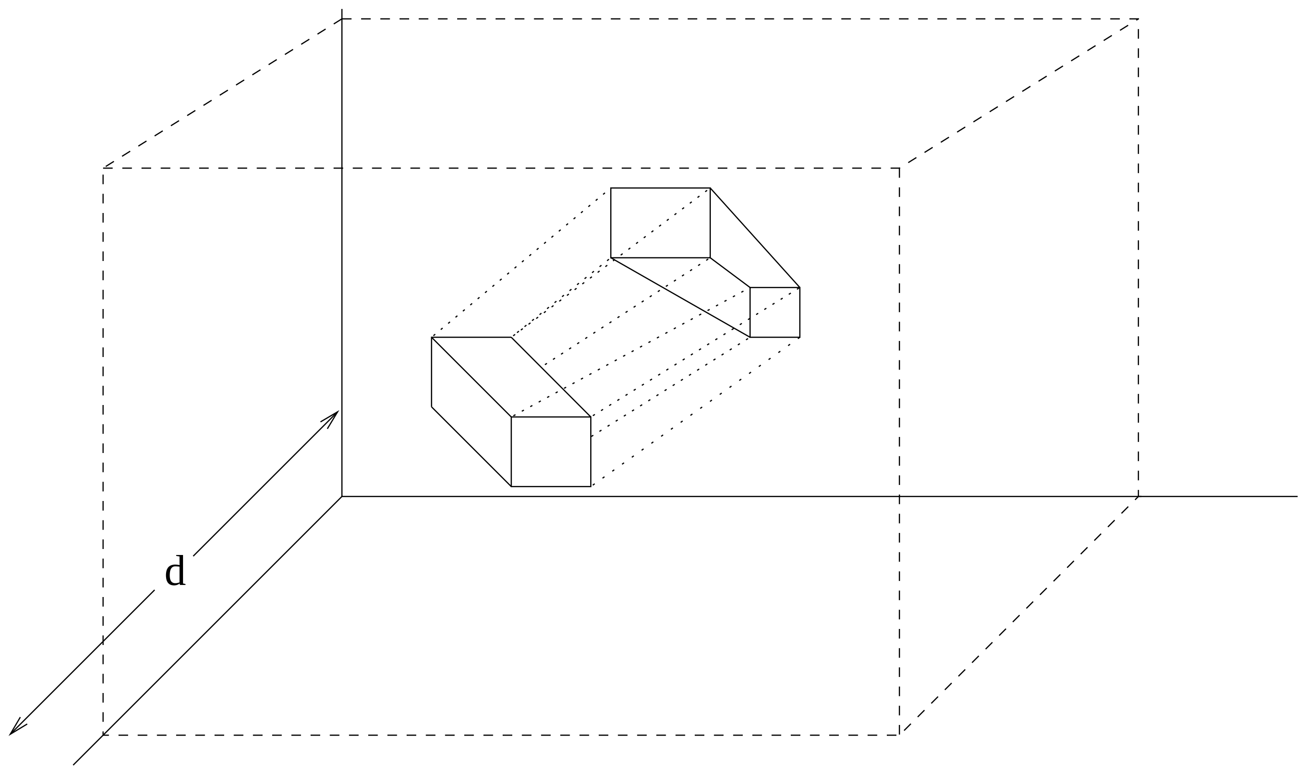
$$\sin^{-1}\left(\frac{\vec{r} \cdot \vec{n}}{|\vec{r}||\vec{n}|}\right) = \theta_{xy}$$

Orthographic Projection



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$

Perspective Projection



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{x \cdot d}{z} \\ \frac{y \cdot d}{z} \\ d \\ 1 \end{pmatrix}$$

Generalized Translation and Rotation

$$Tp = p_T$$

$$\begin{pmatrix} 1 & 0 & \dots & x_{d1} \\ \vdots & \ddots & \vdots & \vdots \\ \vdots & \dots & \ddots & x_{dn} \\ 0 & \dots & \dots & 1 \end{pmatrix} \begin{pmatrix} x_{d1} \\ \vdots \\ x_{dn} \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_{d1} \\ \vdots \\ x_n + x_{dn} \\ 1 \end{pmatrix}$$

$$R_{ij}p = p_R$$

$$\begin{pmatrix} 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & \cos\theta & \dots & -\sin\theta & \ddots & \vdots \\ \vdots & \dots & \dots & \dots & \ddots & \ddots & \vdots \\ \vdots & \dots & \sin\theta & \dots & \cos\theta & \ddots & \vdots \\ \vdots & \dots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_n \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_i \cos\theta - x_j \sin\theta \\ \vdots \\ x_i \sin\theta + x_j \cos\theta \\ \vdots \\ x_n \\ 1 \end{pmatrix}$$

Generalized Projection Matrices

$$\begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & \ldots & \ldots & 1 \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_{n-1} \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & \vdots \\ 0 & \ldots & \ldots & \frac{1}{d} & 0 \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \\ \frac{x_n}{d} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{x_1 \cdot d}{x_n} \\ \vdots \\ \frac{x_{n-1} \cdot d}{x_n} \\ d \\ 1 \end{pmatrix}$$

Future Work: Clipping