ABSTRACT ALGEBRA A
HOMEWORK 1 SOLUTIONS

Chapter 1

1-18. If you just manage not to get confused and do things backwards (not actually a completely trivial task), you get

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \qquad 90^\circ = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$
$$180^\circ = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \qquad 270^\circ = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$
$$d = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix} \qquad a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$
$$v = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \qquad h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

1-19. You could do this with a cardboard square, but you can also work it out algebraically using the machinery from the previous problem:

$$90^\circ \circ d = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = h.$$

1-20. The table looks like this:

$\circ$	e	90°	180°	270°	d	a	v	h
e	e	90°	180°	270°	d	a	v	h
90°	90°	180°	270°	e	h	v	d	a
180°	180°	270°	e	90°	a	d	h	v
270°	270°	e	90°	180°	v	h	a	d
d	d	v	a	h	e	180°	90°	270°
a	a	h	d	v	180°	e	270°	90°
v	v	a	h	d	270°	90°	e	180°
h	h	d	v	a	90°	270°	180°	e

1-21. There is nothing to say here. A binary operation on the set of 8 mappings is just a function from ordered pairs of mappings to the mappings; the table in Problem 1-20 describes the function.

1-22. For no particular reason, here are a few examples:

$$(d \circ a) \circ v = 180^\circ \circ v = h = d \circ 270^\circ = d \circ (a \circ v)$$
$$(90^\circ \circ h) \circ v = a \circ v = 270^\circ = 90^\circ \circ 180^\circ = 90^\circ \circ (h \circ v)$$
$$(180^\circ \circ v) \circ 90^\circ = h \circ 90^\circ = d = 180^\circ \circ a = 180^\circ \circ (v \circ 90^\circ)$$
$$(a \circ 90^\circ) \circ a = h \circ a = 270^\circ = a \circ v = a \circ (90^\circ \circ a)$$
$$(v \circ v) \circ v = e \circ v = v = v \circ e = v \circ (v \circ v)$$

1-23. We've already observed in Problem 1-21 that the composition operation is a binary operation on the set. Theorem 1-3 showed that composition is always associative, saving us a lot of grief. The first column and row of the group table in Problem 1-20 show that e is a 2-sided identity. Finally, there are 2-sided inverses: $90^\circ \circ 270^\circ = 270^\circ \circ 90^\circ = e$; and every other element is its own inverse (look at all those e's on the table's diagonal).

1-24. All it takes is a single example, like the fact that

$$d \circ 90^\circ = v \neq h = 90^\circ \circ d.$$