

CALCULUS A
HOMEWORK 2 SOLUTIONS

SECTION 1.3

1. (a) $y = f(x) + 3$.
(b) $y = f(x) - 3$.
(c) $y = f(x - 3)$.
(d) $y = f(x + 3)$.
(e) $y = -f(x)$.
(f) $y = f(-x)$.
(g) $y = 3f(x)$.
(h) $y = f(x)/3$.
3. (a)=3, (b)=1, (c)=4, (d)=5, (e)=2. It's hard to give reasons beyond the obvious ones.
6. The original graph seems to be doubled in height and slid right a distance 2; so it would be $y = 2f(x - 2)$.
7. This seems to be turned over, slid left 4, and slid down 1; so how about $y = -f(x + 4) - 1$?
27. (a) To the right of the y -axis, the graph of $y = f(|x|)$ and the graph of $y = f(x)$ look exactly alike; to get the part of the graph of $y = f(|x|)$ to the left of the y -axis, take the part of the graph of $y = f(x)$ to the right of the y -axis and flip it over the y -axis to produce an even function.
(b) The graph of $y = \sin |x|$ is in Figure 1.
(c) The graph of $y = \sqrt{|x|}$ is in Figure 2.

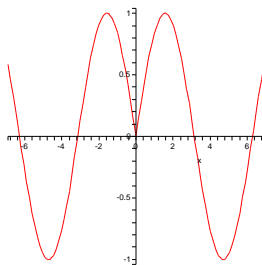
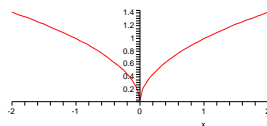
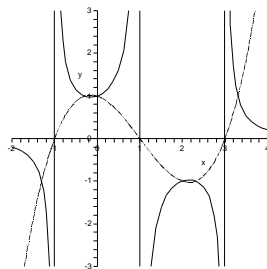


FIGURE 1. $y = \sin |x|$.

FIGURE 2. $y = \sqrt{|x|}$.

28. A sketch is in Figure 3. We pretty well walked through the procedure in class, focusing on the zeros of f , which correspond to the vertical asymptotes of $1/f$. We also looked at places where $f = 1/f = \pm 1$ and places where $f \rightarrow \pm\infty$, so that $1/f \rightarrow 0$.

FIGURE 3. Problem 28: $y = f(x)$ and $y = 1/f(x)$.

37. $(f \circ g)(x) = \sin(1 - \sqrt{x})$. Its domain is $[0, \infty)$.

$$(g \circ f)(x) = 1 - \sqrt{(\sin x)}. \text{ Its domain is}$$

$$\dots \cup [-2\pi, -\pi] \cup [0, \pi] \cup [2\pi, 3\pi] \cup \dots,$$

the set of all points at which $\sin x \geq 0$.

$$(f \circ f)(x) = \sin(\sin x). \text{ Its domain is the set } \mathbb{R} \text{ of all real numbers.}$$

$$(g \circ g)(x) = 1 - \sqrt{1 - \sqrt{x}}. \text{ Its domain is } [0, 1].$$

38. $(f \circ g)(x) = 1 - 3(5x^2 + 3x + 2)$. Its domain is \mathbb{R} .

$$(g \circ f)(x) = 5(1 - 3x)^2 + 3(1 - 3x) + 2. \text{ Its domain is also } \mathbb{R}.$$

$$(f \circ f)(x) = 1 - 3(1 - 3x). \text{ Again the domain is } \mathbb{R}.$$

And $(g \circ g)(x) = 5(5x^2 + 3x + 2)^2 + 3(5x^2 + 3x + 2) + 2$, again with a domain of \mathbb{R} .

39. These are more of a mess—probably worth simplifying so you can see what’s going on.

$$(f \circ g)(x) = g(x) + \frac{1}{g(x)} = \frac{x+1}{x+2} + \frac{x+2}{x+1}.$$

The domain contains all real numbers except -1 and -2 .

$$(g \circ f)(x) = \frac{f(x)+1}{f(x)+2} = \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} = \frac{x^2 + x + 1}{(x+1)^2}.$$

The domain is all real numbers except 0 and -1 . Be sure you think about why I’ve excluded 0 from the domain.

$$(f \circ f)(x) = f(x) + \frac{1}{f(x)} = x + \frac{1}{x} + \frac{1}{x + \frac{1}{x}} = x + \frac{1}{x} + \frac{x}{x^2 + 1}.$$

Every number except 0 is in the domain.

$$(g \circ g)(x) = \frac{g(x)+1}{g(x)+2} = \frac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2} = \frac{2x+3}{3x+5}.$$

Can you see that the domain now includes all real numbers except $-\frac{5}{3}$ and -2 ?

40. Why did I assign so many of these? $(f \circ g)(x) = \sqrt{2x^2 + 5}$, with a domain of \mathbb{R} .
 $(g \circ f)(x) = (\sqrt{2x+3})^2 + 1 = 2x + 4$, but only on the domain of $[-\frac{3}{2}, \infty)$.
 $(f \circ f)(x) = \sqrt{\sqrt{2x+3} + 3}$. The domain is $[-\frac{3}{2}, \infty)$.
 $(g \circ g)(x) = (x^2 + 1)^2 + 1$. The domain is \mathbb{R} .

43. $f(x) = x^{10}$ and $g(x) = x^2 + 1$.

44. $f(x) = \sin x$ and $g(x) = \sqrt{x}$.

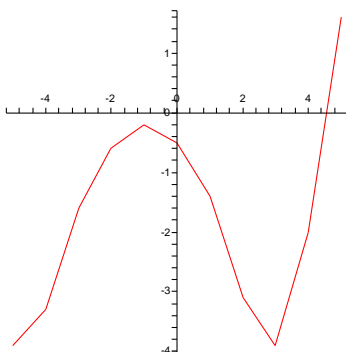
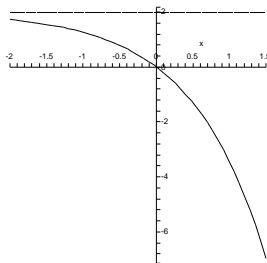
45. $f(x) = \sqrt{x}$ and $g(x) = \cos x$.

46. $f(x) = \frac{x}{1+x}$ and $g(x) = \tan x$.

52. Estimate like this: $f(g(-5)) = f(-0.2) = -3.9$. Similarly, I would approximate $f(g(-4)) = -3.3$, $f(g(-3)) = -1.6$, $f(g(-2)) = -0.6$, $f(g(-1)) = -0.2$, $f(g(0)) = -0.5$, $f(g(1)) = -1.4$, $f(g(2)) = -3.1$, $f(g(3)) = -3.9$, $f(g(4)) = -2$, $f(g(5)) = 1.6$. A graph obtained by just connecting up these points is shown in Figure 4. The real graph is presumably somewhat smoother than this.

SECTION 1.5

12. To get a rough sketch, start with the graph of $y = e^x$. Reflect it across the x -axis to get $y = -e^x$. Move it one unit up to get $y = 1 - e^x$. Finally, stretch vertically by a factor of 2 to get $y = 2(1 - e^x)$ (Of course, there are other ways to get the curve.) The rough graph is in Figure 5.
13. I assume we are to start fresh in each part of the problem. If this is all part of a single process, then you’ll get different and more complicated answers.
 (a) $y = e^x - 2$. (b) $y = e^{x-2}$. (c) $y = -e^x$. (d) $y = e^{-x}$. (e) $y = -e^{-x}$.

FIGURE 4. Problem 52: $(f \circ g)(x)$.FIGURE 5. Problem 12: $y = 2(1 - e^x)$.

18. To have the two points given on the curve, we need to have

$$\begin{aligned} 2 &= f(0) = Ca^0 \\ \frac{2}{9} &= f(2) = Ca^2. \end{aligned}$$

The first of these equations easily solves to give $C = 2$; plugging this into the second equation gives $\frac{1}{9} = a^2$, from which $a = \frac{1}{3}$. We therefore have $f(x) = 2\left(\frac{1}{3}\right)^x$.

Challenge: Why is $a = \frac{1}{3}$ instead of $a = -\frac{1}{3}$?

19. If $f(x) = 5^x$, then $f(x+h) = 5^{x+h} = 5^x 5^h$. Thus,

$$\frac{f(x+h) - f(x)}{h} = \frac{5^{x+h} - 5^x}{h} = \frac{5^x 5^h - 5^x}{h} = \frac{5^x(5^h - 1)}{h} = 5^x \left(\frac{5^h - 1}{h} \right).$$

20. Assuming the month has 30 days, the second payment schedule gives me

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{29} = 2^{30} - 1 = 1073741823 \text{ cents} = \$10,737,418.23.$$

Obviously, I would prefer this scheme. If the month has 31 days, I'd like it even better. Even if the month has only 28 days, the second scheme gives me

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{27} = 2^{28} - 1 = 268435455 \text{ cents} = \$2,684,354.55.$$

Only for months of 26 or fewer days would the first scheme be preferable.

- 22.** There are two points of intersection: one at $x = 1.764921915$, which *Maple* finds easily, and one at $x = 5$, which humans find by looking at it. Two different viewing windows showing most of what goes on with these functions are in Figure 6 and Figure 7.

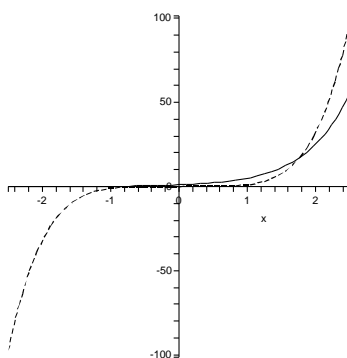


FIGURE 6. Problem 22: $y = x^5$ (dashed) and $y = 5^x$ (solid).

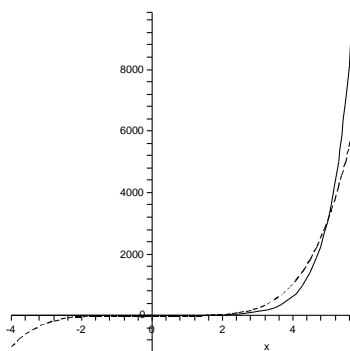


FIGURE 7. Problem 22: $y = x^5$ (dashed) and $y = 5^x$ (solid).

- 25. (a)** There are $100(2^5) = 3200$ bacteria.

- (b) That would be $100(2^{t/3})$ bacteria.
- (c) It would be $100(2^{20/3})$. This is between $100(2^6) = 6400$ and $100(2^7) = 12,800$, closer to the latter than to the former. A more precise estimate would be 10159.36673259647663841091609134277286650555729855905926277302887568.
- (d) The plot is shown in Figure 8. It would appear that the population reaches

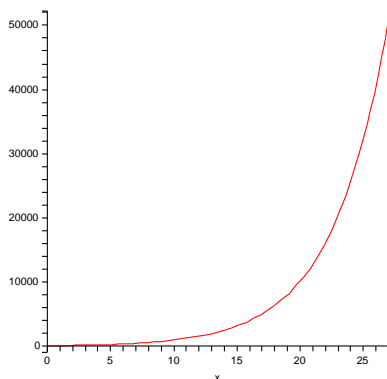


FIGURE 8. Problem 25: $y = 100(2^{t/3})$.

50,000 after not quite 27 hours. Armed with the log function from the next section, one can say that the population is 50,000 when

$$\begin{aligned} 100(2^{x/3}) &= 50,000 \\ 2^{x/3} &= 500 \\ x &= 3 \log_2 500 = 3 \left(\frac{\ln 500}{\ln 2} \right) \doteq 26.8973528539862611308328748. \end{aligned}$$

SECTION 1.6

3. f is not one-to-one because $f(2) = f(6)$.
5. f is one-to-one since every horizontal line hits the graph of f at most once.
9. f is not one-to-one. One could see this by plotting the function and observing that it fails the horizontal line test, which is easy enough to do with a parabola. One could also work algebraically: pick a y , say $y = 0$. We can then solve

$$\begin{aligned} f(x) &= 0 \\ x^2 - 2x &= 0 \\ x(x - 2) &= 0. \end{aligned}$$

This shows that $f(0) = f(2) = 0$; so f is not one-to-one.

25. The usual approach would be to swap x and y and then solve for y :

$$\begin{aligned}x &= \ln(y + 3) \\e^x &= e^{\ln(y+3)} \\e^x &= y + 3 \\e^x - 3 &= y.\end{aligned}$$

Thus, $f^{-1}(x) = e^x - 3$.

28. The inverse of the function $f(x) = 2 - e^x$ should be given by

$$\begin{aligned}x &= 2 - e^y \\e^y &= 2 - x \\y &= \ln(2 - x).\end{aligned}$$

Thus,

$$f^{-1}(x) = \ln(2 - x)$$

In Figure 9, I plot f and f^{-1} . It's worth taking a moment to think a bit about these plots. In particular, where does each graph have an asymptote?

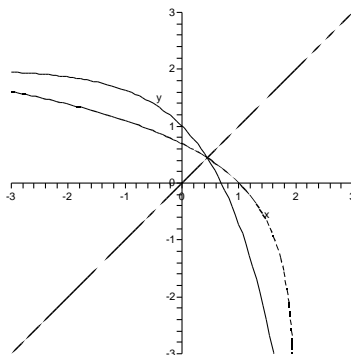


FIGURE 9. Problem 28: $f(x) = 2 - e^x$ and its inverse.

33. (b) $1/36 = 6^{-2}$, so $\log_6(1/36) = -2$.

34. (b) $\ln(e^{\sqrt{2}}) = \sqrt{2}$.

35. (b) Just use facts about logs:

$$\begin{aligned}\log_5(10) + \log_5(20) - 3\log_5(2) &= \log_5(10) + \log_5(20) - \log_5(2^3) \\ &= \log_5\left(\frac{10 \cdot 20}{8}\right) = \log_5(25) = 2.\end{aligned}$$

36. (b) $e^{3\ln 2} = e^{\ln(2^3)} = 2^3 = 8$.

44. This problem is sort of the dual of Problem 20 in Section 1.5. There we saw evidence that any exponential function grows faster than any power of x . Here, we see evidence that any logarithm grows slower than any power of x . There are two points of intersection. *Maple* easily finds the first one near $x = 3.059726680$. The behavior of the two functions near this point is shown in Figure 10.

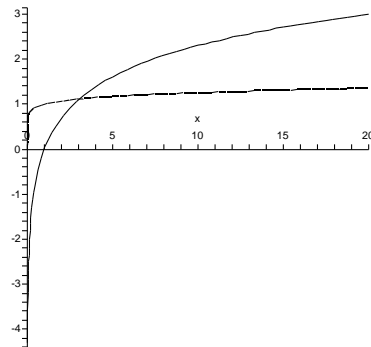


FIGURE 10. Problem 44: $y = \ln x$ (solid) and $y = x^{0.1}$ (dotted).

To find the other point of intersection, one can ask *Maple's* **fsolve** to find a root between 5 and ∞ . *Maple* then delivers up the answer that x is around $3.430631121 \times 10^{15}$. Alternatively, one could have tried out various large values of x : 1,000,000, 10^{100} , $10^{1,000,000}$, etc., trying to find where the crossing point was.

A viewing window showing the second crossing point is shown in Figure 11.

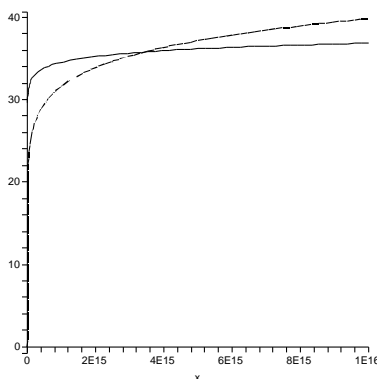


FIGURE 11. Problem 44: $y = \ln x$ (solid) and $y = x^{0.1}$ (dotted).

47. (b) If $e^{-x} = 5$, then $-x = \ln 5$; so $x = -\ln 5$.

48. (b) Similarly, if $\ln(5 - 2x) = -3$, then

$$\begin{aligned} e^{\ln(5-2x)} &= e^{-3} \\ 5 - 2x &= e^{-3} \\ x &= \frac{5 - e^{-3}}{2}. \end{aligned}$$

49. (b) Perhaps the easiest way to think about this one is to use properties of the log functions:

$$\begin{aligned} \ln x + \ln(x - 1) &= 1 \\ \ln(x(x - 1)) &= 1 \\ e^{\ln(x(x-1))} &= e^1 \\ x(x - 1) &= e \\ x^2 - x - e &= 0 \\ x &= \frac{1 \pm \sqrt{1 + 4e}}{2} \\ x &\doteq 2.222870230 \text{ or } -1.222870230. \end{aligned}$$

There's a final gotcha here, though: Only the positive solution makes sense, since you can't take the log of a negative number. You might enjoy thinking carefully about how and where the extraneous negative solution got introduced.

54. You can only take logs of positive numbers, so in order for x to be in the domain, one must have $2 + \ln x > 0$; that is, one must have $x > e^{-2}$.

To get the inverse function, interchange x and y and solve for y :

$$\begin{aligned} x &= \ln(2 + \ln y) \\ e^x &= 2 + \ln y \\ e^x - 2 &= \ln y \\ e^{e^x - 2} &= y. \end{aligned}$$

The inverse function is therefore $f^{-1}(x) = e^{e^x - 2}$. It's worth plotting both f and f^{-1} in order to get some picture of how these functions look.