

CALCULUS A, TEST 2

A large part of the purpose of this test is to make sure you know the formulas and the procedures to compute derivatives and related ideas by hand.

This exam should therefore be done without reference to your book or notes, and without a calculator or Sage.

You need not simplify the results of your calculations. As always in math, you should show me what you are doing, explaining so a reader can understand and follow your work. When I ask for numerical answers, please give me exact expressions. I'm looking for exact answers like e^π , not numerical approximations like 23.140692632779269007.

Please remember that you are writing to be read.

1. Find the equation (not just the slope) of the tangent line to the curve $y = x^3 + 4x^2 - 7$ at the point $(1, -2)$.

2. (a) State a definition of the derivative in terms of limits. Explain this definition with a sketch, labeled to show what's going on, and perhaps with a few words.

- (b) Use this definition and nothing else (like the Power Rule) to compute $f'(1)$ when $f(x) = x^3 + 2x$.

3. Compute the derivatives of the following functions of x . Do not bother to simplify your answers.

(a) x^e .

(b) e^x .

(c) e^e .

(d) x^x .

(e) $x^{-7} - 3\sqrt{x} + \frac{5}{x}$.

(f) $\sqrt{x \cos x}$.

(g) $\frac{x^5 e^x}{7x^3 + 5x}$.

(h) $2^{\log_2 x}$. (*Hint*: Thinking a moment is your friend.)

4. Work out the following derivatives:

(a) $\left(\frac{g(\sqrt{x^2+1})}{\sqrt{x^2+1}}\right)'$.

(b) The second derivative $\frac{d^2}{dx^2}(f(x)g(x))$.

(c) $\frac{d^2}{dx^2}(f(g(x)))$.

5. Suppose you know that $f(2) = 0.3$ and that $f'(2) = -3$.
- (a) What's your best guess for the location of a root of f (a point where $f(x) = 0$)?
 - (b) How would your guess change (at least qualitatively) if you also knew that $f''(2) = 1$?

6. Figures 1 and 2 show the plot of $y = f(x)$. Give approximate sketches of the graphs of $y = f'(x)$, $y = f''(x)$, and an antiderivative for $f(x)$.

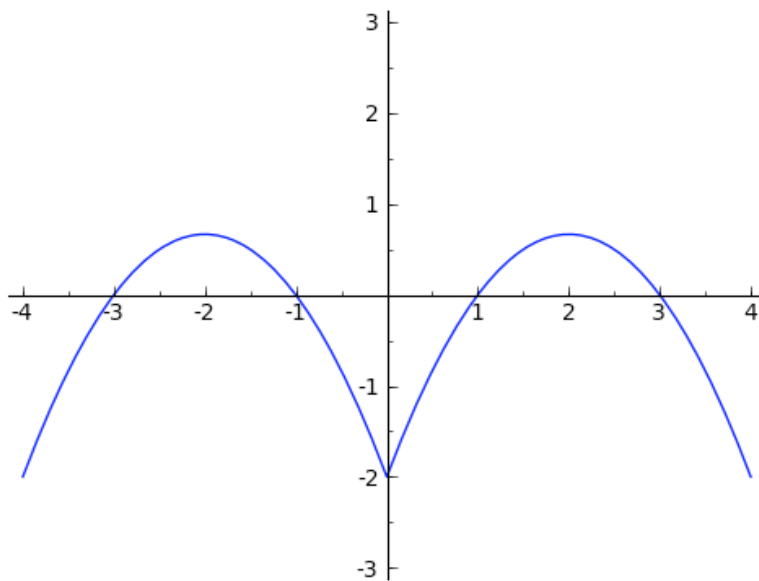


FIGURE 1. Problem 6: $y = f(x)$. Plot $y = f'(x)$ and $y = f''(x)$ here.

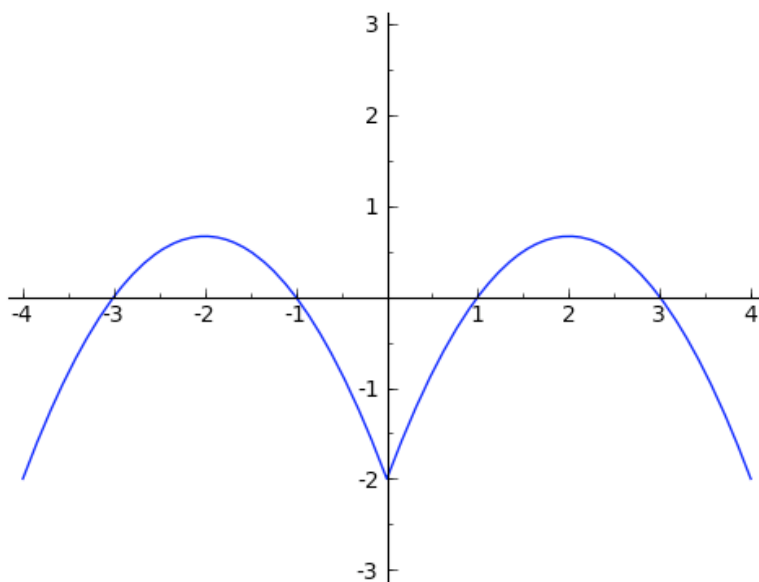


FIGURE 2. Problem 6: $y = f(x)$. Plot an antiderivative for $f(x)$ here.

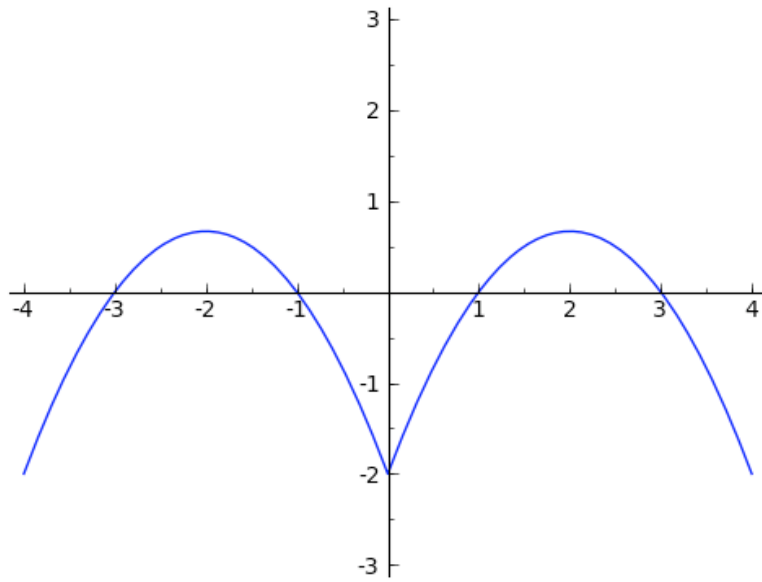


FIGURE 3. Problem 6: $y = f(x)$. An extra plot if you need it.