DISCRETE MATHEMATICS
HOMEWORK 10

The first lot of problems here are inspired by problems in Kenneth Rosen's *Discrete Mathematics and its Applications.*

1. Suppose I uncharacteristically give you a multiple-choice test with 10 questions, each having 5 possible answers. How many possible ways could you fill out this test assuming you are required to give exactly one answer to every question? What if you are allowed to leave some questions blank?

2. How many strings of 3 English letters are there? How many strings of three distinct English letters (i.e., without repetition)?

3. How many license plate numbers are there with 6 characters total consisting of 1 or more letters followed by digits? (That is, possible plate numbers might include EC1847 and TIMMCL.)

4. I realize that there are English words in which Y and W are vowels. (*Cwm* is the obvious choice for W.) Let's make life easy for ourselves, though, and define A, E, I, O, U as vowels and the other 21 letters as consonants. Count the number of strings of 8 English letters satisfying each of the following conditions.

(a) All letters are consonants, and no letter can be repeated.
(b) All letters are consonants, and repeated letters are allowed.
(c) The first letter is a vowel, and no letter can be repeated.
(d) The first letter is a vowel, and repeated letters are allowed.
(e) At least one letter is a vowel, and no letter can be repeated.
(f) At least one letter is a vowel, and repeated letters are allowed.
(g) The first letter is a W, and at least one letter is a vowel, and no letter can be repeated.
(h) The first letter is a W, and at least one letter is a vowel, and repeated letters are allowed.

5. Here's a question on politics and leadership.

(a) At Squirrel'em College, 25 students show up to an All-Student Meeting. How many ways can 4 of these students be selected to be the co-convenors of the student body?
(b) At another college, there are 25 students on the Executive Committee of the Student Senate. How many ways can students be chosen from this committee to fill the 4 offices of Prime Minister, Chancellor of the Exchequer, Minister of War, and Commissar of Internal Security?

6. For a fund-raiser, 100 tickets numbered 1–100 are sold. Four distinct numbers are then selected to win the four prizes: Third Prize of breakfast for one at Saga, Second Prize of lunch for one at Saga, First Prize of dinner for one at Saga, and a Grand Prize of a two-night stay for two at Muncie's prestigious Motel 6. Assume, as is usual in these drawings, that each ticket wins at most one prize. How many ways are there to award the prizes with each of the following constraints:

(a) There are no restrictions?
(b) Ticket 47 wins the Grand Prize.
(c) Ticket 47 wins one of the prizes.
(d) Ticket 47 doesn't win any of the prizes.
(e) Tickets 19 and 47 both win prizes.
(f) Tickets 19, 23 and 47 all win prizes.
(g) Tickets 19, 23, 35 and 47 all win prizes.
(h) None of tickets 19, 23, 35 and 47 win prizes.
(i) The Grand Prize goes to either ticket 19, 23, 35 or 47.
(j) Tickets 19 and 23 win prizes, and tickets 35 and 47 do not.

7. The parts of this problem vary significantly in simplicity. Do as many as you can. Be creative!

Suppose you have 4 children: Peter, Nicholas, Joanna, and Nina. You also have 5 fruits: an apple, a banana, a cucumber, a date, and an eggplant. Finally, you have 5 identical candy bars.

(a) In how many ways can you distribute the 5 fruits among the 4 kids if you are under no restrictions about how many of the fruits each kid gets? (You can be as fair as possible, or you can give all the fruit to Joanna, for instance.)
(b) How many ways can you distribute the fruit if you are required to give at least 1 fruit to each kid?
(c) How many ways can you give the candy bars out to the kids if you can be as unfair as you like? Notice that this problem is different from part (a), since now the candy bars are all alike. A kid might prefer a banana and a date to a cucumber and an eggplant, but two candy bars are two candy bars.
(d) How many ways can you give out the candy bars if each kid has to get at least 1?