

DISCRETE MATHEMATICS
HOMEWORK 12

1. Count the number of distinct rearrangements of the letters in the following words:
 - (a) BOTHER
 - (b) HASSLE
 - (c) ANNOYANCE
 - (d) MISSISSIPPI

2. Suppose we have a set of 7 people: Adam, Britney, Cain, Delilah, Eve, Fred, and Ginger.
 - (a) How many ways can 5 of these people be picked out to make a kids' soccer team?
 - (b) How many ways can we pick out a team of one goalie and 4 other players?
 - (c) How many ways can we pick out a team of 5 players and then line them up for a photograph?
 - (d) How many ways can we line up all 7 people?
 - (e) How many ways can we line up all 7 people and have Adam next to Eve?
 - (f) How many ways can we line up all 7 people and have Adam not next to Eve?

3. A party consists of 8 men and 8 women.
 - (a) How many ways can we divide people into pairs to dance if, for reasons of decency, each pair must consist of one man and one woman?
 - (b) How many ways can we divide people into pairs to dance if, as payback for part (a), each pair must consist either of two men or two women?
 - (c) How many ways can we divide people into pairs to dance if, for a radical idea, we decide to make no restrictions as to who may dance with whom?

4. A department contains 10 men and 15 women.
 - (a) How many ways are there to choose a committee of 6 from this department if, for fairness, the committee must contain an equal number of women and men?
 - (b) How many ways are there to choose a committee of 6 from this department if, for fairness, the committee must contain more women than men?

5. Show that for any n and k ,

$$\binom{n+2}{k} = \binom{n}{k} + 2\binom{n}{k-1} + \binom{n}{k-2}.$$

This could be done either combinatorially or algebraically, or perhaps by other methods.

6. Show either combinatorially or by algebraic manipulation:
 - (a) $k\binom{n}{k} = n\binom{n-1}{k-1}$.
 - (b) $(n-k)\binom{n}{k} = n\binom{n-1}{k}$.
 - (c) $\binom{n}{k}\binom{k}{r} = \binom{n}{r}\binom{n-r}{k-r}$.

7. Suppose you have 4 children: Peter, Nicholas, Joanna, and Nina. You also have 5 fruits: an apple, a banana, a cumquat, a date, and an eggplant. Finally, you have 5 identical candy bars.
- (a) In how many ways can you distribute the 5 fruits among the 4 kids if you are under no restrictions about how many of the fruits each kid gets? (You can be as fair as possible, or you can give all the fruit to Joanna, for instance.)
 - (b) How many ways can you distribute the fruit if you are required to give at least 1 fruit to each kid?
 - (c) How many ways can you give the candy bars out to the kids if you can be as unfair as you like? Notice that this problem is different from part (a), since now the candy bars are all alike. A kid might prefer a banana and a date to a cumquat and an eggplant, but two candy bars are two candy bars.
 - (d) How many ways can you give out the candy bars if each kid has to get at least 1?
8. What are the elements of the following sets:
- (a) $\{1, 2, 3\} \cup \{1, 3, 5\}$.
 - (b) $\{1, 2, 3\} \cap \{1, 3, 5\}$.
 - (c) $\{1, 2, 3\} \setminus \{1, 3, 5\}$.
9. Find all elements and all subsets of the following sets:
- (a) \emptyset .
 - (b) $\{\emptyset\}$.
 - (c) $\{1, \emptyset\}$.
 - (d) $\{\emptyset, \{\emptyset\}\}$.