

DISCRETE MATHEMATICS
HOMEWORK 12

1. A department contains 10 men and 15 women.
 - (a) How many ways are there to choose a committee of 6 from this department if, for fairness, the committee must contain an equal number of women and men?
 - (b) How many ways are there to choose a committee of 6 from this department if, for fairness, the committee must contain more women than men?

2. Show that for any n and k ,

$$\binom{n+2}{k} = \binom{n}{k} + 2\binom{n}{k-1} + \binom{n}{k-2}.$$

This could be done either combinatorially or algebraically, or perhaps by other methods.

3. Show either combinatorially or by algebraic manipulation:

- (a) $k\binom{n}{k} = n\binom{n-1}{k-1}$.
- (b) $(n-k)\binom{n}{k} = n\binom{n-1}{k}$.
- (c) $\binom{n}{k}\binom{k}{r} = \binom{n}{r}\binom{n-r}{k-r}$.

4. A party consists of 8 men and 8 women.

- (a) How many ways can we divide people into pairs to dance if, for reasons of decency, each pair must consist of one man and one woman?
- (b) How many ways can we divide people into pairs to dance if, as payback for part (a), each pair must consist either of two men or two women?
- (c) How many ways can we divide people into pairs to dance if, for a radical idea, we decide to make no restrictions as to who may dance with whom?

5. Once again, we have four children: Peter, Nicholas, Joanna, and Nina.

- (a) In how many ways can 7 cats, all differently marked, be distributed among 4 children, if each child could get anywhere from 0 to 7 cats? *Remark:* Attempts to curry favor by trying this experiment (or any of the ones below) using Tim's children are not guaranteed to succeed.
- (b) In how many ways can 7 identical and indistinguishable goldfish be distributed among the 4 children?
- (c) What if we split the difference and try to count the number of ways to distribute among the 4 children 3 indistinguishable goldfish and 3 indistinguishable catfish? One can tell a goldfish from a catfish, but all goldfish look alike, as do all catfish.
- (d) As a very special gift, a student decides to distribute among 4 children 7 custom drawn tattoos, all different. For simplicity, all the tattoos go on the children's left arms. A child getting more than one tattoo has the tattoos arranged in a straight line going up the arm. The order matters. Thus, arrangements where Nina is given the anchor, the Harley logo, and the smiling Quaker are different from arrangements where Nina gets

the smiling Quaker, the anchor, and the Harley logo. In how many ways could this gift be given? Can you even begin to imagine what would happen next?

(A simpler way to phrase this problem is, in how many ways can you place 7 distinguishable flags on 4 distinguishable flagpoles, if the order of the flags on each flagpole matters?)

- (e) Repeat part (a) with the added condition that each child must get at least 1 cat.
- (f) Repeat part (b) with the added condition that each child must get at least 1 goldfish.