

DISCRETE MATHEMATICS
HOMEWORK 2

1. This problem is about the question of which fractions exist in \mathbb{Z}_m .

(a) Find the fraction $\frac{1}{2}$, if it exists, in each of the arithmetics

$$\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{11}, \mathbb{Z}_{12}, \mathbb{Z}_{13}.$$

Can you conjecture a general rule about which clock arithmetics contain $\frac{1}{2}$?
Can you conjecture a general rule for how to compute $\frac{1}{2}$ if it exists? Can you convince me of your conjectures?

(b) Do the same thing with $\frac{1}{3}$.

(c) Do the same thing with $\frac{1}{4}$. Do your findings agree with what you learned in parts (a) and (b)?

(d) Try to find $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ in \mathbb{Z}_{10} . Comment, conjecture, convince.

2. The function $\sigma(n)$ is the sum of the positive divisors of n . For instance, $\sigma(7) = 8$, since the positive divisors of 7 (1 and 7) add up to 8. Similarly, $\sigma(10) = 18$, since the positive divisors of 10 (1, 2, 5, and 10) add up to 18; and $\sigma(9) = 1 + 3 + 9 = 13$.

Try to find a formula for $\sigma(n)$ in terms of the prime factorization of n . That is, if

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k},$$

then

$$\sigma(n) = \sigma(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = ?$$

3. Convince me of the truth or falsehood of the assertion that if $a \mid b$ and $b \mid c$, then $a \mid c$.