DISCRETE MATHEMATICS
HOMEWORK 3

In several of the problems in this set, we'll be solving equations like $3x - 10y = 1$. In all cases, we are interested only in solutions where both x and y are integers.

1. Find at least one solution to the equation $3x - 10y = 1$ in which x and y are both integers. Find more than 1 solution (or all solutions?) if you can.

2. Do the same thing with each of the following equations. Note: some of these equations may not have any integer solutions. If you can't find any solutions, then try to see if you can give an argument why no solutions could exist.

(a) $x - 10y = 1$.
(b) $2x - 10y = 1$.
(c) $4x - 10y = 1$.
(d) $5x - 10y = 1$.
(e) $6x - 10y = 1$.
(f) $7x - 10y = 1$.
(g) $8x - 10y = 1$.
(h) $9x - 10y = 1$.

What do Problems 1 and 2 have to do with Homework 2, Problem 1(d)?

3. Find these greatest common divisors:

(a) $\gcd(30, 216)$.
(b) $\gcd(125, 725)$.
(c) $\gcd(-9, 0)$.
(d) $\gcd(100, 543546984630)$.
(e) $\gcd(12345678, 987654321)$.

4. Here's a last one on number theoretic functions. Two numbers a and b are said to be *relatively prime* if they share no common divisors except ± 1, i.e., if $\gcd(a, b) = 1$. For instance, 4 and 15 are relatively prime, even though neither 4 nor 15 is prime.

The function $\phi(n)$ counts the number of positive integers $a \leq n$ such that a and n are relatively prime. For instance, $\phi(7) = 6$, since 7 is relatively prime to the 6 numbers 1, 2, 3, 4, 5, and 6. Similarly, $\phi(10) = 4$, since the only positive integers less than 10 with which 10 is relatively prime are the 4 numbers 1, 3, 7, and 9.

Try to find a formula for $\phi(n)$ in terms of the prime factorization of n. That is, if

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k},$$

then

$$\phi(n) = \phi(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = ?$$

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