DISCRETE MATHEMATICS
HOMEWORK 7

1. Find a number x which simultaneously solves the two congruences

$$x \equiv 6 \pmod{10}$$
$$x \equiv 4 \pmod{7},$$

or convince me that there is no such x. Can you find all numbers x which solve these congruences?

Repeat this exercise with the two congruences

$$x \equiv 6 \pmod{12}$$
$$x \equiv 11 \pmod{21}.$$

2. Try to convince me that congruences work pretty much like equalities. In particular, argue that

(a) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $(a+c) \equiv (b+d) \pmod{m}$.
(b) If $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{m}$.
(c) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.

3. Compute the following numbers in $\mathbb{Z}_{37}$. Try to be as efficient as you can in your arithmetic. You ought to be able to do these by hand. If you use a machine, then discuss how you would do the calculations with pencil and paper.

(a) 5^2.
(b) 5^4.
(c) 5^8.
(d) 5^{16}.
(e) 5^{32}.
(f) 5^{17}.
(g) 5^{36}.
(h) 5^{22}.

4. Use Fermat's test to convince me that 39 is not a prime. (Let me be clear: I'd really like to to use the Fermat test, even though it is not the quickest way to see that 39 is composite.)

5. Compute the sums

$$1$$
$$1+3$$
$$1+3+5$$
$$1+3+5+7$$
$$\text{etc.}$$

until you see a pattern. Can you convince me of the truth of this pattern?

6. Compute the sums

$$2^0$$
$$2^0+2^1$$
$$2^0+2^1+2^2$$
$$2^0+2^1+2^2+2^3$$
$$\text{etc.}$$

until you see a pattern. Can you convince me of the truth of this pattern?