3 Calc A, Lab 3

In this lab, I'd like to explore both graphically and algebraically the process of estimating the slopes of tangent lines to curves, and to tie this in with limits and derivatives. Again, please work in pairs if you can. My reading of previous labs certainly seems to suggest that one actually learns by discussing problems with others.

As always, please explain your reasoning, and please write to be read. Use Maple where it helps, and avoid it where it hinders.

Let f be the function given by the rule $f(x) = \sqrt{25 - x^2/4}$. We'll be doing a lot with this function, so it might make sense to define it in Maple by doing something like

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f := x -> sqrt(25-x^2/4);
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We'll be interested in the function f, and in the slope of the tangent line to $y = f(x)$ at $x = -8$, that is, at the point $(-8, 3)$.

1. What is the domain of f?

2. Make a large and careful plot of the graph of f, and sketch (I'd do it by hand) the tangent line to the graph at the point $x = -8$. Estimate the slope of this line from the graph. Please be principled and stick to your estimate once you have made it.

3. Find the y coordinate of the point on the graph of f with $x = 10$. Sketch the line joining this point to our original point $(-8, 3)$, and compute the slope of this line. Repeat this process with the line connecting $(-8, 3)$ to the point on the graph with $x = 6$, the line connecting $(-8, 3)$ to the point on the graph with $x = 2$, the line connecting $(-8, 3)$ to the point on the graph with $x = -2$, and the line connecting $(-8, 3)$ to the point on the graph with $x = -6$. Which of these lines best approximates the tangent line at the point $(-8, 3)$?

4. Write an equation for the slope of the line joining the point $(-8, 3)$ to the point at $(x, f(x))$. Be as explicit as you can.

5. Use your observations in (3), together with any other calculations you might like to do, in order to get as accurate an estimate as you can for the slope of the tangent line at $(-8, 3)$. How well does this estimate agree with what you conjectured graphically in (2)?

6. Once you have the slope of the tangent line, write the equation of this line. Plot the line and the original function on the same axes. Do they look tangent?

7. Write down a limit whose value is the slope of the tangent line to $y = f(x)$ at the point $(-8, 3)$. Evaluate this limit analytically.

8. In class, we've defined the slope of the tangent line to $y = f(x)$ at the point $(x, f(x))$ as the derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Use this formula to compute the derivative of $y = x^3$. Plot y and y' on the same graph. Does it make sense that the slope of y should be the height of y'?

9. Repeat this with $y = x^4$. Can you conjecture anything about the derivative of x^n for any m? Can you argue for your conjecture?

10. Repeat problem 8 with the function $y = x^3 - 3x$. Discuss what features of the graph of y can be read off from the graph of y' and vice-versa.