FIVE-YEAR UNIT REVIEW
DEPARTMENT OF MATHEMATICS
EARLHAM COLLEGE

Goals

A. Our unit's goals have not changed since those cited in the 1992–1993 NCA accreditation self-study.

B. We do not anticipate changes in those unit goals.

Having said this, we should perhaps add that as a result of the current discussion of General Education requirements, there may be changes in the *College's* goals regarding quantitative skills as a subject of General Education. These changes, should they occur, will probably have rather minimal impact on the Math Department's goals or curriculum. We are already teaching as many Gen Ed fulfilling classes as we can teach while still maintaining a major. Our most popular Gen Ed course, and the one closest in spirit to the proposed Quantitative Literacy requirement, is Elementary Statistics, which has typical enrollments of about 40 per section. We would be extremely reluctant to see these sections increase in size, and it is impossible to increase the number of sections while maintaining the major. Should the College develop a goal of quantitative literacy, therefore, the vast bulk of the curricular impact of that decision will be seen in quantitative courses outside the Mathematics Department.

C. Plans for meeting goals are always changing. At the same time, the current curriculum of the Department could largely be described by saying that we teach those courses without which we could not meet our service commitments, or without which we could not offer a credible major. That being so, our freedom to propose significant changes in our curriculum is rather tightly constrained. The following, somewhat speculative answers (which are, after all, answers to questions asking not what we propose, but what we think we might propose over the next 5 years), may give an idea of what curricular ideas the Department is currently exploring.

(1) We are not now proposing major revisions of any existing courses.

(2) There are a number of courses we wish we could offer, and that we would like to tweak the curriculum to allow. We're not yet sure whether any amount of tweaking can make this possible.

Our highest priority course to add would be a regular course in geometry. Although in modern times, geometry (except in quite sophisticated forms like algebraic geometry, arithmetic geometry, and topology) has rather fallen out of fashion, geometry seems to us to be an essential course for a mathematics department at a liberal arts college. Not only is geometry one of the 7 liberal arts (so that in principle, one course in seven at the College should be in geometry), but geometry

has been at the heart of Western mathematics during most of the $2\frac{1}{2}$ millennia it has been practiced as a science. Geometry stands at the heart of the axiomatic enterprise that has shaped mathematics since the Greeks. The development in the past 2 centuries of a rich variety of non-Euclidean geometries marks the beginning of a mathematically and philosophically deep process of reexamination of that axiomatic approach. Finally, geometry is a highly useful course for our majors looking to pursue careers in elementary or secondary education.

Another area in which we would like to be able to augment our offerings is statistics. We currently teach Elementary Statistics, and we have an Intermediate Statistics course on the books; but we have no plans to offer Intermediate Stats in the foreseeable future. Given the number of students and Departments who depend on statistics, this is a great pity. Among the schools of the GLCA, only Antioch has statistical offerings as thin as Earlham's (but then, this is largely true in all areas of mathematics, as the attached spreadsheet shows).

A third course we wish we could offer is a seminar for first or second year students with an interest in mathematics. We think that such a course would help prospective majors to become part of the wider mathematical community. It would also help students to understand early in their careers what mathematics is about—which is not, for the most part, what mathematics in the high schools is about, so that students often arrive at college confused. An early seminar would be a useful asset in helping students make the transition from computational mathematics to real mathematics based on proof.

We have also explored whether it might be possible and beneficial to split the two sections of Calculus A we offer in the fall into two separate tracks for students with varying levels of mathematical maturity, as half the GLCA schools do.

Finally, we'd like to offer a wider range of upper level courses on an occasional basis: topology, number theory, mathematical modeling, dynamical systems, and symbolic and algebraic computation all come to mind as possibilities.

The problem, of course, comes in trying to envision whether there is anything we could drop in order to take up this bounty. It is difficult to see that there is any course we currently teach that we could completely eliminate. There are, however, courses we might offer less frequently. Perhaps Algebra A and Analysis A need not be offered annually, as they now are, but could be given on an alternate year basis, as Algebra B and Analysis B now are. This would probably entail lowering the major requirements so that not both these courses were required. If demand for our courses changes at the end of our revision of Gen Ed requirements, it is possible we might be able to eliminate one section of Calc A or of Statistics each year, but this is far from a sure thing.

There are other more exotic solutions, like fusing DE and Linear Algebra; but the impact of this on Physics might be serious, and in any case, since DE is currently being taught by a physicist, this wouldn't buy us anything.

As a consequence, many of the ideas above will likely not make the transition to reality.

(3) As long as we don't eliminate any courses, it's hard to see that any of the changes we are toying with would have significant impacts outside the Department. Should we reach the point of splitting Calculus A into multiple tracks, we would obviously

consult the major consumers of calculus—the other Departments in the Natural Sciences plus Economics. We are not yet at a point of initiating this conversation.

(4) Our last revision of the major tightened it up substantially, mandating that students take both Analysis A and Algebra A, and that they take at least one Analysis B and Algebra B. The resulting major is probably adequate—though well short of optimal—for graduate study. It is one of the more directive majors in the GLCA, though this partly reflects our smaller number of courses relative to other GLCA schools. (In general, our major and all the majors at the GLCA schools are similar.)

We are currently contemplating slightly loosening the requirements for the major, in order to make life easier for those who are not first and foremost mathematicians to do an undergraduate major in the field. The exact details of this change—perhaps requiring only one of Analysis A and Algebra A—is not yet clear. Nor is it clear how we would continue to encourage those wishing to do further work in mathematics after Earlham to do far more than a minimal major. This might be done by advising, or it might be better to offer explicitly more than one major, something along the lines of Pure Math and Math for Applications. This matter is still under discussion; our fellow GLCA institutions offer a wide variety of possible models.

(5) We don't at this point anticipate doing another hire until Mic retires; and it is not easy to think about what our strategy should be for a hire we hope to be more than a decade off. Could we have anticipated in 1990, for instance, that the World Wide Web would be invented, and that most job searching would now be done over the web?

In any case, our current strategy does not seem to be too badly broken. Our last two hires have been women. It is, unfortunately, not surprising that we have not attracted new hires from the lamentably tiny pool of African-American PhDs in mathematics.

We have managed to interest Bob Hunter in teaching statistics for us on at least an occasional basis, and we would love to see that regularized; but this depends on funding issues outside the Department's control. It also appears at this writing that Bob's decisions about his own life are moving him in directions other than teaching mathematics, as much as we might hope he could see his way clear to put our desires ahead of his own. Should Bob's plans change, we'd love to have him here as often as we can; his presence can only help in the effort to broaden the appeal of mathematics.

D. This has largely been addressed already in Section C(2), but a bit more can be said. First, let us try to describe the current state of Earlham's mathematical offerings.

A summary of the mathematics courses listed in the catalogues of the GLCA schools is given in the attached spreadsheet. As can be seen, Earlham offers fewer math courses than any GLCA school except Antioch. In part, this may be because we have largely eliminated from the catalogue courses that are not taught on at least an alternate year basis, but only one of our competitors (Kenyon) seems to list infrequently taught courses in their catalogue. In part, it may also be due to Earlham's size; but Earlham is not by far the smallest GLCA school.

No matter how one tries to rationalize the facts, though, one must face the conclusion

that compared to the colleges of the GLCA, Earlham's math offerings are just very thin.

We offer less than half the number of statistics classes of the GLCA *average,* even if you include Intermediate Statistics, the one course in our Department's catalogue that is never offered. If we remove this fictitious course, only Antioch is as statistically impoverished as Earlham. In the areas of geometry, history of mathematics, mathematical economics, math education, dynamical systems, operations research, mathematical modeling, numerical analysis, topology, complex analysis, number theory, and combinatorics, the *average* GLCA school lists a total of 5.3 courses in its catalogue. Earlham, alone in the GLCA, lists 0. The average GLCA school lists 2.3 courses at a pre-calculus level. Earlham and Kalamazoo are the only schools to have none. (There is a pre-calculus course offered by IU East on the Earlham campus, but Earlham enrollment in this class has been under 10 students, which does not seem representative of the actual level of need for remedial courses among our students.)

Every current member of the Earlham Math Department is in the slightly discouraging situation of having specialized in an area of mathematics that is not regularly taught at the College.

Earlham is slightly richer than the GLCA average in a few areas, most notably in logic; but this is an accident of the composition of the Philosophy Department, whose members teach both our logic courses.

Given the resources to make it possible, we would like to be able to offer as rich a mathematics curriculum as the other schools of the GLCA. This would include

(a) The geometry course mentioned above.
(b) More options in calculus sequences.
(c) More upper level math classes, so that both students and faculty could follow their passions instead of being limited to the same handful of courses. Wouldn't it be nice if we could offer something in some of the areas in the long list a couple of paragraphs back? It wouldn't take much to make our major way more varied and way more exciting than it is at present. We obviously cannot and should not aspire to be Princeton or Chicago; it is a pity, though, that we cannot aspire to be Wabash or Hope.
(d) Something to offer students in need of a course in college algebra or finite mathematics or pre-calculus. The majority of Earlham students never take a math class at college. Many of those are very timid about their abilities within mathematics. It's a pity we have nothing to offer them except the IU East Pre-calculus course currently offered at Earlham, and that we cannot offer them anything without cutting further into what are already the thinnest upper level offerings in the GLCA outside Antioch.

E. One issue has already been mentioned: it is entirely possible that the College will soon approve a requirement of some level of quantitative literacy. Since currently, most Earlham students do not take math at college, this will entail a large increase in enrollment in classes with a mathematical component. It is not completely clear where these classes will be, but it is clear that they can't be in the Math Department. The Department certainly believes that quantitative literacy is important. The notion that someone could graduate from college—especially one devoted to the liberal arts—with no exposure whatever to mathematics boggles the mind. We have to be clear, however, that the Math

Department is currently doing all it can to build quantitative literacy, and that in the absence of additional staffing—which it would be laughable to advocate in the current financial situation—we can't double or triple our enrollments. If there is to be a quantitative literacy requirement, then most students will be meeting that requirement outside our Department. The College can't commit for our Department to meet a need of universal quantitative literacy, because we just can't do it. We don't have the staffing. We'd hate for there to be any misunderstanding on this point.

In fact, staffing is the Department's single largest issue and challenge, as it has been for a long time. We obviously can't expect that to change any time soon, if it ever changes; but we would be derelict in assessing ourselves if we didn't point out the gravity of the situation.

In 1982, the NCA described Earlham's Math Department as presenting a "puzzling picture," and commented, "The team advises that this department is apparently understaffed for the kind of liberal arts college Earlham professes to be. Clearly, the mathematics faculty is overworked and students probably benefit insufficiently from their mathematics training." Since 1982, the Department's staffing has remained essentially static; it is now slightly smaller than in 1982 because of Mic's work in Environmental Programs.

The attached spreadsheet makes it appear that Earlham has the smallest Mathematics Department in the GLCA, except, perhaps, for Antioch, which does not list faculty on its web page. Even if one cuts in half the Departments that also include Computer Science (which probably overstates the CS contribution to those Departments) every GLCA school lists more math faculty than Earlham. Our position at the bottom of the GLCA in course variety, and our almost unique position in not offering any sort of pre-calculus mathematics or any statistics beyond a first course, therefore make sense. We don't have many courses because we don't have many faculty. That's obviously not going to change, but it is a central issue for us. We can offer a credible, if minimal, major, and we can meet the most crying needs of the many programs and students we serve; but there is a great deal we simply cannot do to address the needs of those with weak mathematical skills and to make our major exciting and vibrant. We can't do these things because we don't have the people.

There is another, related, issue that Mathematics faces, again related to staffing and to the richness of our program. Under terms, Math faculty taught 7 courses a year, rather than the standard 6. In moving to a semester calendar, the College mandated a standard teaching load which was the same in all Departments. This change was a source of great joy to us. To be able to teach the same number of hours as our colleagues has made a difference in our lives. It has also cost us richness, since we've had to figure out how to produce a major after, in essence, a $\frac{1}{2}$ FTE cut. Of course, every Department could have a more exciting curriculum if its faculty were each to teach an extra course each year without extra pay. That realization just hits us little harder because we used to do that, and we remember the richness of the former curriculum. The challenge now is to try to capture as much of we can of that former freedom and excitement—and it will not be all of it—without giving in to the temptation to do so on an overload basis and to recapture as well the former bondage and exhaustion.

The Department also has opportunities. If the College does approve a quantitative literacy requirement, we hope it will involve the Department in at least a consultative relationship with the many faculty in many Departments and Divisions who are teaching

the quantitative classes students will use to meet that requirement. We're excited about the possibility of trying to make math look alive and important to students by showing them that math meets them where they live, in problems that matter to them.

We're excited about the small curricular changes we talk about above, even though we're not sure they'll be possible.

We continue to be challenged and excited by the transforming effect computer algebra systems are having on our ways of learning and doing mathematics.

We're excited by the opportunities we have to collaborate with the CS program as it grows in strength and numbers, even as we are challenged by the fact that some of that growth is at our expense.

We're excited about the possibilities offered by Earlham's environmental programs, among others, to do interesting things in mathematical modeling.

Finally, we are excited to live at a moment when most of the mathematicians who have ever lived are currently alive, and when so much deep and beautiful mathematics happens all the time. For mathematicians, as for members of every other discipline, the pace of modern scholarship both threatens and stimulates.

Assessment

A. There have not been changes in our goals over this period.

B. As noted in our last 5 year review, We have made a number of changes on our evaluation procedure since our 1992–3 study. In 1992, the Department of Mathematics proposed assessing the quality of our majors first and foremost by maintaining close contract with them, and by remaining interested in their lives. Given the number of majors our department produces ($\sim$ 5/yr), this seems an entirely sensible strategy. In addition, the department suggested graduate school success, contact with graduates after 5 years, and performance on the comprehensive exams as valuable markers of our success.

At that time, the comprehensive exams were administered by examiners from outside Earlham. We spoke in our accreditation report of improving the comps as a tool for evaluation and information-gathering by providing the examiners with more constraints than we customarily had done. Instead, the Department began in 1994 to prepare the comps in-house. In this way, we could tailor the exams to our own curriculum, and see more accurately how we are doing in areas we regard as central to our educational objectives.

Since then, we have also begun to require that each math major take two credits in the Seminar; and inspired by Tekla Lewin, all of the math faculty have begun attending the seminar each week. The contents of the seminar vary from week to week, including student presentations on particular problems of interest, and more focused study sessions for the comps. These may take the form either of direct study of problems from previous comprehensive exams, or of student-led overviews given area of mathematics, like "What central results from Linear Algebra should every kid know?"

As a result of these procedural changes, we are looking closely at our students in the areas that matter most to us by preparing our own exams. We are also all sitting together each week and watching our students organize and present the material they have learned at Earlham, and that they are reviewing for the comps. This has been a tremendous

opportunity to get to know our students' strengths and weaknesses, and the strengths and weaknesses of our program and our teaching methods. It's interesting how often seminar has been immediately followed by an informal Department meeting in which we say, "I wonder if, in teaching ... we should do ... ."

An additional part of our assessment efforts this time around have been devoted to looking at higher level organizational issues. Are we offering the right blend of courses? Are we correctly structuring the major? The ability the World Wide Web gives us to look quickly at the programs of kindred institutions has offered a wonderful tool for us to learn from the decisions and insights of kindred institutions.

We have not repeated the experiment of sending questionnaires to recent majors, feeling that the previous small sample size and rate of return rendered the value of the data we collected incommensurate with the burden the questionnaires imposed upon us and upon our respondents.

C. Our examination of the comprehensive exams and our experience in the seminar shows progress in dealing with some of our concerns from 5 years ago, though some of the same weaknesses remain. Our students plainly have reasonable computational facility in courses required for the major. Since more courses are now required for the major than was the case 5 years ago, the written comps now cover more advanced and theoretical classes than they used to. As a consequence, the written comps now also let us examine our students' facility in crafting proofs and in understanding the central abstractions of the discipline. As a group, we think they are performing these higher-level tasks well; we also wish they could do them better.

On a higher level still, that of organizing in their minds the central ideas from entire courses, or of seeing how the contents of different courses combine to form a coherent body of knowledge, our best students have some success, and more typical students struggle. It is clear that this sort of integration is not for most of them a natural, autochthonous activity. Where we have prodded and encouraged this in seminar, however, we have been pleased at their ability to produce really quite competent overviews of large bodies of material.

We wish we had yet been able to find ways to inspire students to reflect on their own (rather than after an explicit assignment from us) on the organization and central ideas of their discipline, and to analyse their own process of discovery and verification. We cannot pretend that they are finished scholars; we wish they were closer.

This year's seminar has functioned much more productively than some past years', in part because we have learned to be more directive in assigning integrative tasks. We still find our students' passivity in this enterprise distressing, and we wish we could overcome it. We also are seeing a negative consequence of our focus on evaluating students in settings of freedom, where they can take time, be creative, and make full use of the information they have learned in class. To a distressing degree, students are not troubling themselves actually to memorize very much. We have no desire to turn our courses into experiences in rote memorization, but we need to find ways to impress upon our students that one's effectiveness as the practitioner of any art depends in part upon what one actually knows, and not just upon what one can create and infer and look up.

As was the case 5 years ago, most of our graduates are doing productive work, roughly half of them in areas somewhat related to their math major. A fair number go on to

graduate school in math-related areas, and those students have done well. Only one student from the Department has attended graduate school in mathematics per se in recent years; he seems to have been well thought of by his professors before his death (which, just to be clear, had nothing to do with his mathematical life). He and other students who have studied mathematics in off-campus programs while at Earlham have done well, though not sensationally, in those studies.

Another point of assessment that should perhaps be listed here rather than in section (F) below is that Earlham teams have begun participating in mathematics competitions with those from other institutions. Most notably, an Earlham team in the International Competition in Mathematical Modeling three years ago was one of two teams in the world (the other was from China) to be rated as Outstanding in the problem they undertook. Our other teams in this and other contests have held their own without being quite so dazzling. This year, for instance, we fielded two teams, both of which won Honorable Mentions.

All this suggests that while there are things we should improve, we are equipping our students adequately.

If we further compare Earlham's array of mathematics courses and our major with those of our competitors, we see a more mixed picture, most of which has been spelled out above. To summarize, our major requirements are roughly in line with everyone else's. The range and depth of our curriculum, while adequate for those students with adequate skills to start into our intro classes, is not competitive within the GLCA. We have never been able to offer remedial courses that many of our students would need should they ever elect to become quantitatively literate.

D. In view of the discussion in Section II(C), we are currently reexamining our process of giving comprehensive exams. Where this will lead is not yet clear. We are exploring whether we can learn from Departments with obviously successful comps like Biology, which seems to have a very well thought-out system of announcing parts of the exam in advance, compelling students to learn this material without further help from the faculty, and using outside examiners in better ways than we did when we employed such examiners. We're also trying to explore ways to expand the integrative experience that now happens in one weekend into something that is spread across a student's Earlham career. All of this is in a very preliminary stage right now, but we're excited and serious about it.

The comps were last modified about 7 years ago, when we began writing them in-house and when we began teaching and requiring the Seminar. Changes in the courses required for the major at the point we moved to semesters have also slightly changed the coverage of the written comps, as discussed above.

E. There have not been changes in our goals over this period.

F. We frankly do not have much that is not of an anecdotal nature. Students come up to us after class to report that mathematics has always frightened them, but that now they feel more confident about it, that they begin to see how one thinks about it, and that they want to study more. They say the same thing in course evaluations. Helping even a few people to feel this way is already a sizable success, exactly in the area in which success is most important to us. Of course, others fail to catch fire, and one always struggles to be

both clearer and more inspiring; and of course, not all of our colleagues are convinced we optimally prepare students for the mathematical needs of other disciplines.

It is also worth pointing out that we have had some wonderful successes with students majoring in other disciplines who were serious students of mathematics as well. Biology students have done wonderful independent study projects with us in dynamical systems, and one of these students has gone on to become a prominent ecologist in Australia. The students whose team did so splendidly in the mathematical modeling competition a few years back were both non-majors in the Department. These serious and excited consumers of mathematics from other disciplines seem to us to be among our clearest and most pleasing success stories.

As was the case 5 years ago, one of the great questions we face is how to serve those who never take a math class. Do we serve them better by leaving them alone to live out their lives in ignorance, or by compelling them to do more math? Much of this remains unclear; we are still debating Gen Ed requirements. It is clear that the staffing of the Mathematics Department will not allow us departmentally to do more than we are doing now to address the needs of these students; we hope that the College can find some means to address them.

G. We do not have plans at this point to modify our procedures. What will happen at the end of our discussion of General Education is at this point impossible to predict.

H. We think hard and discuss our curriculum and our teaching with one another, and we try to do better. This process has improved significantly since Jennifer finally induced us to hold weekly Departmental Meetings, which we had never previously held. Our collective presence at the Seminar and at the weekly Mathophiles seminar further fosters the exchange of ideas.

I. We can think harder and discuss more effectively.

Grants

The Department has no proposals currently in the works, nor are we currently receiving grants. We do not have a wish-list of proposals.

Equipment

The equipment we need to keep going are calculators and computer algebra systems, plus the means to display their results to classes. To maintain our current situation (which is adequate), we will need

(a) Continued licensing by the College of *Maple*® or its equivalent.
(b) Continued licensing by the College of *SPSS*® or its equivalent.
(c) Continued availability of teaching labs like D224.
(d) Continued availability of computers with large video monitors or with projection units.
(e) Continued availability of projection units for calculators.

(f) Although compared with PCs, the graphing calculator market is relatively static and mature, at some point our existing collection of TI-92 calculators will become obsolete, and we will need either to replace them, to ask students to buy their own calculators, or to revert to computers.

There are times at which it would be more convenient for us if we had more computers attached to some sort of video display viewable by a classroom. We currently need to wheel large, unwieldy carts from room to room in order to use *Maple*® or *SPSS*® in class, which means that we often do not have them handy when we want them spur-of-the-moment. Fixing this, however, would require large expense for rather low-use devices, which is probably not cost-effective unless other Departments here need the equipment as well.

A more important place where improved equipment is needed is in the computing support area. Some of us have stopped using the D224 lab in class because so many of the machines are often not working; and all of us have been frustrated by this. The installation of *Maple*® here and in other labs on campus is often not correct, frustrating students as well. The folks in ECS are working hard trying to get situations like these under control, but they are too few people chasing too many problems. The result is that the College has paid to buy labs that not all faculty will use because we don't have the resources to keep them working. This is hard to feel good about.

We also need chalk and blackboards, and some of us need coffee. All of us need sleep.

Foreign Workers

We have not used outside consultants of any sort in preparing this document.