CHALLENGE PROBLEM II

Tim McLarnan

This week's Challenge Problem looks awful, but gives a very exciting result with much less work than you'd think. The expression

$$Q(n) = 1 + \sum_{m=1}^{2^n} \left\lfloor \left(\frac{n}{\sum_{j=1}^{m} \left\lfloor \cos^2 \left(\pi \frac{(j-1)!+1}{j} \right) \right\rfloor} \right)^{1/n} \right\rfloor$$

computes a well-known function. What function is it?

The function $\lfloor x \rfloor$, which appears twice in this formula, is the integer part, or floor, of x. It is x rounded down to the nearest integer. For example, $\lfloor 3.14 \rfloor = \lfloor 3.94 \rfloor = \lfloor 3 \rfloor = 3$.

Hint: Compute a few values of $Q(n)$ by computer (using *Maple*?).

Hint^2*:* Try working from the inside. What function is computed by

$$\left\lfloor \cos^2 \left(\pi \frac{(j-1)!+1}{j} \right) \right\rfloor ? \qquad \sum_{j=1}^{m} \left\lfloor \cos^2 \left(\pi \frac{(j-1)!+1}{j} \right) \right\rfloor ?$$

Hint^3*:* You may find yourself discovering Wilson's Theorem, or remembering it, or looking for it in number theory books.

Tim McLarnan will award a cool prize to the student submitting the best solution of this problem received on or before October 23.

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As before, faculty may participate for the fun and glory, but must buy cool prizes for themselves.