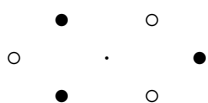


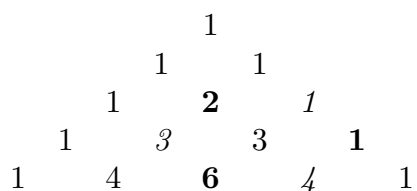
## A MATHEMATICAL OFFERING, PROBLEM 4

TIM MCLARNAN

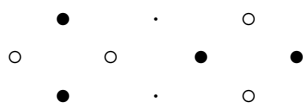
The pattern



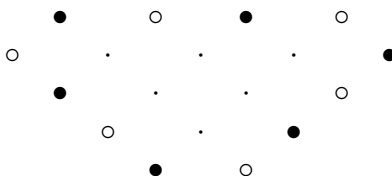
is called a *perfect square pattern* (PSP) in Pascal's triangle. This means that wherever this little hexagon is placed in Pascal's triangle, the product of the entries marked  $\bullet$  equals the product of the entries marked  $\circ$ . For example, in the triangle below, the product  $\mathbf{2} \cdot \mathbf{6} \cdot \mathbf{1}$  of the bold entries equals the product  $\mathit{1} \cdot \mathit{3} \cdot \mathit{4}$  of the italic entries.



Two other PSPs are this one



and this monster



- Prove that each of these patterns really is a PSP.
- Find as many other PSPs as you can.
- Find a necessary and sufficient condition for a pattern to be a PSP.

It might be helpful to know that I learned this problem as an undergraduate from Zalman Usiskin of the University of Chicago, who is a mathematics educator very interested in transformational geometry—geometry done by focusing on the translations, rotations, and dilations of figures. A very geometrical solution exists.

On a beautiful problem like this, I'll give you 2 weeks to think.

Tim McLarnan will award a cool prize to the student submitting the best solution of this problem received on or before November 20.

As always, F is for “Faculty” and “Fun,” while P is for “Pupil” and “Prize.”