

## A MATHEMATICAL OFFERING, PROBLEM 6

TIM MCLARNAN

Here's a final question to reflect on for this semester.

- (1) If  $2^n - 1$  is a prime, what can you say about  $n$ ?
- (2) If  $2^n + 1$  is a prime, what can you say about  $n$ ?
- (3) What if both  $2^n - 1$  and  $2^n + 1$  are primes?

*Remark:* Your solutions to problems (1) and (2) will be partial. There is no known necessary and sufficient condition for  $n$  in either problem (1) or (2). In both cases, primality implies simple facts about  $n$ ; but these simple facts about  $n$  are not enough by themselves to imply primality.

If you want to gather numerical data, the *Maple* command `isprime()` might be useful. It is a quick, probabilistic primality test. If you haven't seen probabilistic prime tests, you might want to poke about in books on number theory or discrete math, or to grab a math prof. It's beautiful stuff.

As always, Tim McLarnan will award a cool prize to the student submitting the best solution of this problem received

Typeset by  $\mathcal{A}\mathcal{M}\mathcal{S}$ - $\mathcal{T}\mathcal{E}\mathcal{X}$

on or before Reading Day. This would be a lovely way to get started on your holiday shopping.