

DISCRETE MATHEMATICS
HOMEWORK 1

Here are a few problems to think about between now and Friday, and to write up between now and Monday. Try to think creatively about the questions, and to convince the class of your answers.

1. Let the system \mathbb{Z}_5 denote the “clock arithmetic” consisting of the numbers 0, 1, 2, 3, 4. Arithmetic is done just like they taught you in grade school, except that whenever the answer is 5 or greater, you subtract off copies of 5 until it is small enough. For instance, $2 + 2 = 4$, but $3 \cdot 4 = 12 = 12 - 5 = 7 = 7 - 5 = 2$.

Find the following elements of \mathbb{Z}_5 , if they exist: -1 , -2 , $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\sqrt{2}$, $\sqrt{-1}$. Do the same thing in \mathbb{Z}_7 and in \mathbb{Z}_6 . Can you say anything about these numbers in \mathbb{Z}_n for an arbitrary integer n ?

2. Suppose you know about the integers (the whole numbers), but you don't know much else about mathematics. You want to talk about when one integer divides another. You decide to use the notation $a \mid b$ to mean “ a divides b ” (i.e., “ b is a multiple of a ”). The symbol $a \mid b$ is not a numerical value, but a true-false assertion. For instance, $3 \mid 6$ is a true statement, and $3 \mid 5$ is a false statement.
- (a) Give a definition of $a \mid b$ in mathematical terms.
 - (b) Convince me of the truth or falsehood of the assertion that for all integers a , b , c , if $a \mid b$, then $a \mid bc$.
 - (c) Convince me of the truth or falsehood of the assertion that for all integers a , b , c , if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.
 - (d) Convince me of the truth or falsehood of the assertion that for all integers a , b , c , if $a \mid bc$, then $a \mid b$ or $a \mid c$.

3. We write numbers using 10 digits 0–9 because we have 10 fingers. Presumably lobsters, whose hands (well, claws) have only 2 parts each would write numbers in base 4, using only the digits 0–3. Thus, for us, the number after 9 is 10, meaning 1 ten and 0 ones. For a lobster, the number after 3 is 10, meaning 1 four and 0 ones. We write numbers in base 10, while the lobsters use base 4.

When different bases are in use, one sometimes uses a subscript to indicate which base. Thus, the number of original states in the US (thirteen) could be written as

$$(13)_{10} = (31)_4,$$

since

$$(13)_{10} = 1(10) + 3 = 3(4) + 1 = (31)_4.$$

- (a) Write the numbers from one to thirty (i.e., $(30)_{10}$) in base 4.
- (b) Think about a good way to convert numbers from base 4 to base 10, and write $(3133)_4$ in base 10.
- (c) Think about a good way to convert numbers from base 10 to base 4, and write $(107)_{10}$ in base 4.