DISCRETE MATHEMATICS
HOMEWORK 17

This homework is the last of the 5 larger assignments or take-home tests which collectively count for 60% of your grade. Please take it seriously; please don't take it too seriously. You are welcome to use your notes. You are welcome to use computing devices. Please don't use other books, and please don't discuss the test with other people.

I've tried to make the test clear and accurate, but nothing in this life comes with a guarantee. Feel free to call me at work (x-1351) or at home (966-0520, but not after 9PM, please), or to send e-mail to `timm` if you have questions. If I find errors on the test, I will send e-mail to the class; so looking at your mail now and then might be wise. It is always sensible to make sure early on that you understand the questions.

This homework is to be turned in no later than Wednesday, December 19. If I'm not in my office, you may leave it with Mary Lou Rosser, the Math Dept. Secretary, or you may put it in my mailbox. Keep a copy if you're at all nervous about whether your paper will get to me.

Please continue to remember that in mathematics, no answer is complete without an explanation. I am much less interested in learning that the number of possible threesomes of US congresspersons is 13,624,345, than in learning where this number came from, and why it is correct. Please don't give numerical answers to questions without adding some quick sentence or formula to justify your claims. Also, please remember that the different parts of the test differ substantially and capriciously in difficulty. Stay calm. Finally, understand that I am perfectly content to see the numerical parts of answers written in forms like

$$\binom{435}{3} = \frac{435 \cdot 434 \cdot 433}{3!}$$

rather than in forms like 13,624,345. Don't bother to multiply out answers unless you want to.

Good luck; do well; learn something new; have as much fun as possible.

1. A party is planned for 3 married couples, each consisting of one woman and one man. They are to be seated in a row along one side of a long table.

(a) How many seating arrangements total are possible?
(b) In how many seating arrangements do men and women alternate?
(c) In how many seating arrangements is each person adjacent to their spouse?
(d) How many seating arrangements have men and women alternating, and have each person adjacent to their spouse?
(e) How many seating arrangements have men and women alternating, and have no person adjacent to their spouse?

2. Let the Flubbonacci numbers be defined by the relations

$$\begin{aligned} f_0 &= 2 \\ f_1 &= 1 \\ f_{n+1} &= f_n + 2f_{n-1}, \quad \text{if } n > 0. \end{aligned}$$

Show that the Flubbonacci numbers are given by the formula

$$f_n = 2^n + (-1)^n.$$

3. The Sports Section:

(a) The game of wallyball is played by a team of 5 people: a forward, a backward, a guard, a lard, and a half-full. Play is highly specialized, so each player plays only one position. How many ways can 5 people be chosen out of our class of 22 and assigned positions to form a wallyball team?

(b) The game of polyball is played by a team of 5 people, who just mill about together on the field, nobody doing anything special. How many ways can 5 people be chosen out of our class of 22 to form a polyball team?

(c) Finally, the game of Baliball is a less specialized version of wallyball. It is also played by a team of 5 people, two of whom (the forward and backward) play specialized positions, and the other three of which are all lards. How many ways can 5 people be chosen out of our class of 22 and assigned positions on a Baliball team?

4. In how many ways can 12 balls be distributed to 4 distinguishable bags if

(a) The balls are distinguishable?

(b) The balls are indistinguishable?

(c) The balls are distinguishable and each bag gets at least 1 ball?

(d) The balls are indistinguishable and each bag gets at least 1 ball?

(e) The balls are distinguishable and each bag gets 3 balls?

(f) The balls are indistinguishable and each bag gets 3 balls?

5. Repeat the previous exercise in the case the 4 bags are indistinguishable.

6. The binomial coefficients section:

(a) Prove that for any integer $n > 1$,

$$\binom{n+1}{3} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \cdots + \binom{n}{2}.$$

I could imagine doing this combinatorially, or by induction. Can you generalize this result?

(d) Show either algebraically or combinatorially that

$$\binom{n}{k}\binom{n-k}{s} = \binom{n}{k+s}\binom{k+s}{k}.$$