CALCULUS A, TAKE-HOME TEST 2

In this part of the exam, you may use your book and notes (including my online homework solutions) as well as the computing device(s) of your choice. You may not use other sources like other books, Google, Wikipedia, your sister the Princeton professor, or Joe the Plumber. Obviously, you need to work alone and not to talk to others about the test. The smart way to do that is not to talk to people about math at all between now and class Wednesday. When I ask for numerical answers, please give me exact expressions, simplified as far as possible. I'm looking for exact answers like e^π, not numerical approximations like 23.140692632779269007. As always, explain what you're doing and please remember that you are writing to be read. Obviously I'm expecting your solutions to be on other pieces of paper, not wedged in the white spaces on this page.

This part of the test is due in class on Wednesday. I am almost completely intolerant of late tests.

1. Use the definition of the derivative in terms of limits, and nothing more sophisticated, to compute $f'(x)$, where $f(x) = \dfrac{1}{x^2 + x}$.

2. Show how to compute by hand as economically as possible the following quantities:

(a) $r'(x)$, where $r(x) = ((x^{10} + x)^{10} + x)^{10} + x$.

(b) $\dfrac{d}{dx}\left(\ln\left(\sin^4(f(x)) \cdot g(x)^{10} \cdot \sqrt{h(x)} \cdot e^{(e^x)}\right)\right)$.

(c) $s'(x)$ where $s(x) = e^{(e^{(e^x)})}$.

(d) The slope of the tangent line to the curve given by $x^3y^2 - xy^5 = 6$ at the point $(2, 1)$.

3. The table below shows the values of two functions at a few points.

x	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	4.177	2.850	2.236	2.279	3.000	3.621	3.764	3.150	1.823
$g(x)$	1.482	1.775	2.000	1.894	2.356	2.507	2.646	2.775	2.897

(a) Estimate as accurately as possible $g'(1)$.
(b) Estimate as accurately as possible $g''(1)$.
(c) Estimate as accurately as possible $(d/dx)(f(g(x)))$ at $x = 1$.

4. Suppose you know that $f(2) = 0.3$ and that $f'(2) = -3$.

(a) What's your best guess for the location of a root of f?
(b) How would your guess change qualitatively if you also knew that $f''(2) = 1$?
(c) Can you make an argument for a new quantitative guess if $f''(2) = 1$?

5. Figures 1 and 2 show the plot of $y = f(x)$. Give approximate sketches of the graphs of $y = f'(x)$, $y = f''(x)$, and an antiderivative for $f(x)$.

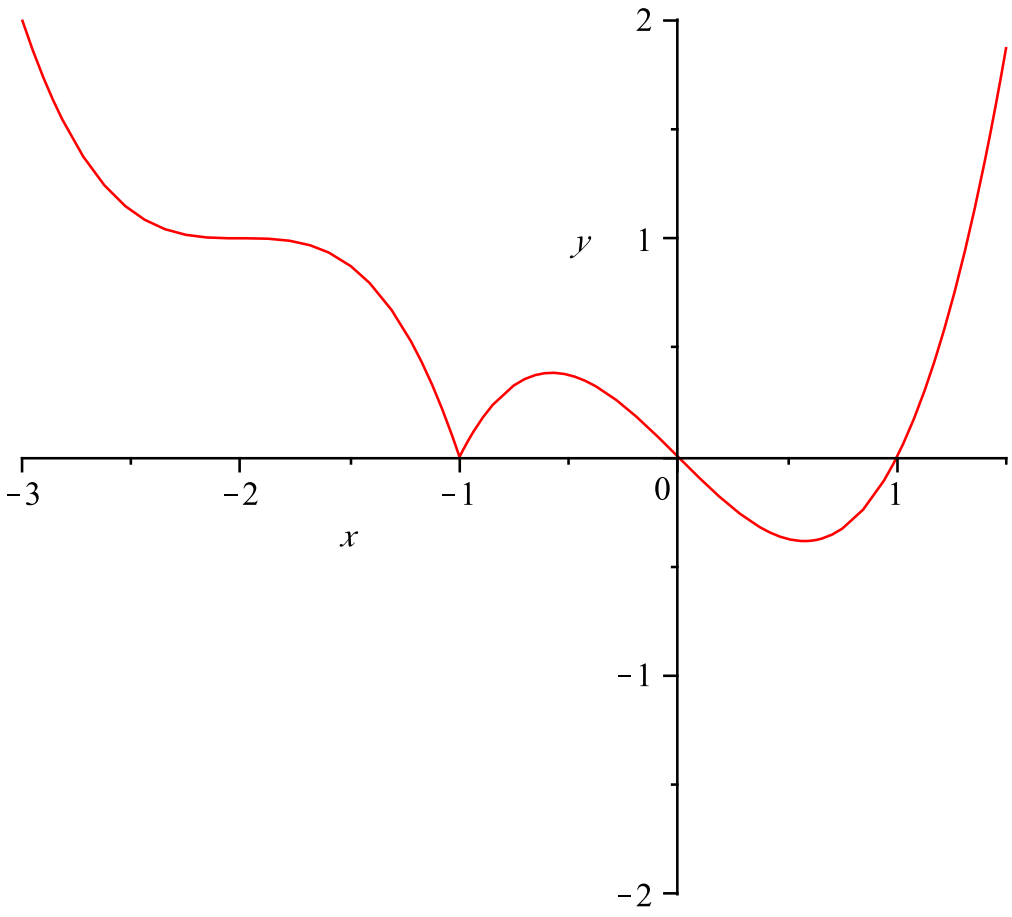

FIGURE 1. Problem 5: $y = f(x)$. Plot $y = f'(x)$ and $y = f''(x)$ here.

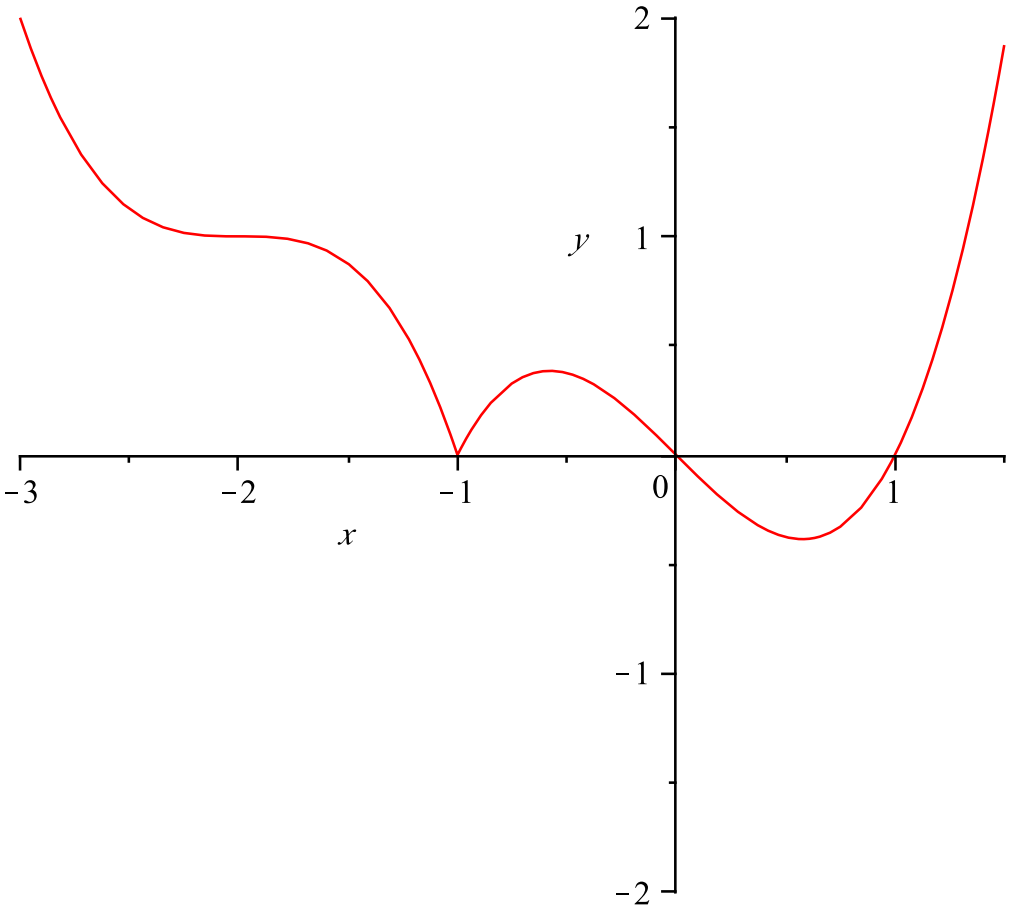

FIGURE 2. Problem 5: $y = f(x)$. Plot an antiderivative for $f(x)$ here.