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CALCULUS A
TEST 1 MAKEUP

This exam is closed book and closed notes, and is to be done by hand. On the bright side, I'm pretty sure a calculator would be useless on it in any case. When I ask for numerical answers, please give me exact expressions, simplified as far as possible. I'm looking for exact answers like e^π, not numerical approximations like 23.140692632779269007.

Please write your answers clearly, remembering that I can only give you points for what I can read and understand.

1. What's the equation of the line through the points $(2, 6)$ and $(5, 8)$? What's its slope?

The slope is $\frac{8-6}{5-2} = \frac{2}{3}$; so the equation is $y-6 = \frac{2}{3}(x-2)$, or

$y = \frac{2}{3}x + \frac{14}{3}$

2. Let $f(x) = x^2\sqrt{x-1}$.

(a) Compute $f(3x)$. You need not simplify.

$f(3x) = (3x)^2\sqrt{3x-1}$

(b) Compute $f(x+3)$. You need not simplify.

$f(x+3) = (x+3)^2\sqrt{(x+3)-1}$

3. Solve for x by hand, and show your work or explain your answer.

(a) $x = \log_8(1/2)$.

$\frac{1}{2} = \frac{1}{\sqrt[3]{8}} = \frac{1}{8^{1/3}} = 8^{-\frac{1}{3}}$; so $\log_8 \frac{1}{2} = -\frac{1}{3}$

(b) $x = \log_4(8)$.

$8 = 4\sqrt{4} = 4 \cdot 4^{1/2} = 4^{3/2}$; so $\log_4 8 = \frac{3}{2}$

(c) $\log_2 x + \log_2(x+1) = 0$. (Are there 2 solutions, or fewer?)

$$\log_2(x(x+1)) = 0 \Rightarrow x(x+1) = 2^0$$
$$\Rightarrow x^2 + x = 1$$
$$\Rightarrow x^2 + x - 1 = 0$$
$$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

Are both these points solutions, though?
No, because $x = \frac{-1-\sqrt{5}}{2}$ forces us in the original equation to take $\log_2\left(\frac{-1-\sqrt{5}}{2}\right)$, which is impossible. So only $x = \frac{-1+\sqrt{5}}{2}$ is actually a solution, one which works in the original equation.

4. Use the techniques of Chapter 2 to compute the following limits exactly by hand, showing your work. I'm looking for exact results, not probable approximations.

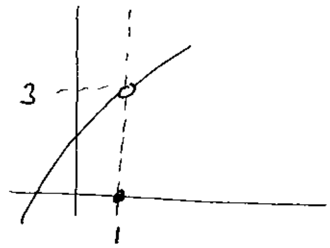
(a) $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$. $= \lim_{x \to 3} \frac{(x-3)(x+1)}{(x-3)(x-2)} = \lim_{x \to 3} \frac{x+1}{x-2} = 4$

Factoring the numerator and denominator is not too hard, since you know one of the factors will be x-3.

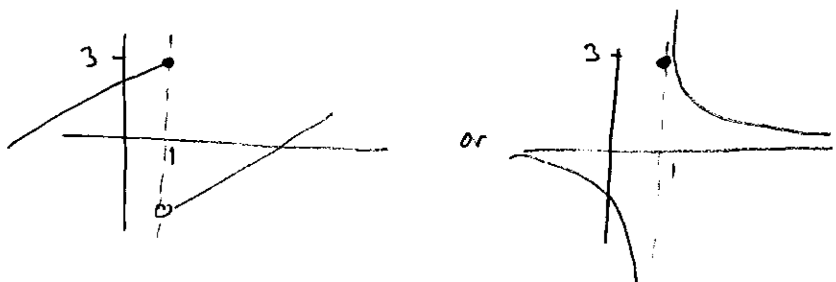
(b) $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$. $= \lim_{x \to 2} \frac{1}{x-2} \left[\frac{1}{x} - \frac{1}{2} \right] = \lim_{x \to 2} \frac{1}{x-2} \cdot \frac{2-x}{2x} = \lim_{x \to 2} \frac{-1}{2x} = -\frac{1}{4}$.

5. Make quick sketches of graphs of the following functions, or explain why they do not exist. You need not have formulas for your functions. All functions should be defined for every real number x except perhaps for $x = 1$.

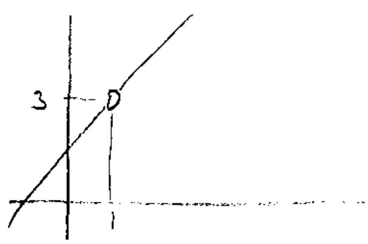
(a) A function a for which $\lim_{x\to 1} a(x) = 3$, but $a(1) = 0$.



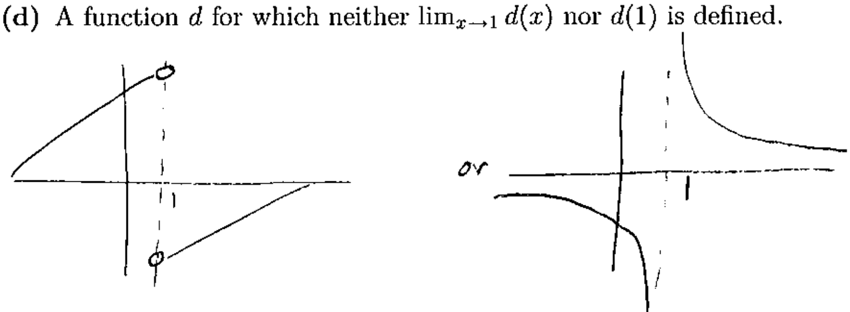
(b) A function b for which $b(1) = 3$ but $\lim_{x\to 1} b(x)$ is undefined.



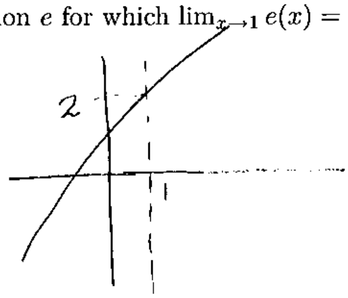
(c) A function c for which $\lim_{x\to 1} c(x) = 3$, but $c(1)$ is undefined.



(d) A function d for which neither $\lim_{x\to 1} d(x)$ nor $d(1)$ is defined.



(e) A function e for which $\lim_{x\to 1} e(x) = e(1) = 2$.



6. Figure 2 shows the graph of the functions $f(x)$. Make quick sketches of the graphs of the following functions. The attached grids might be good places to do this.

(a) $f(2x)$.
(b) $f(x+1)$.
(c) $-2f(x)$.

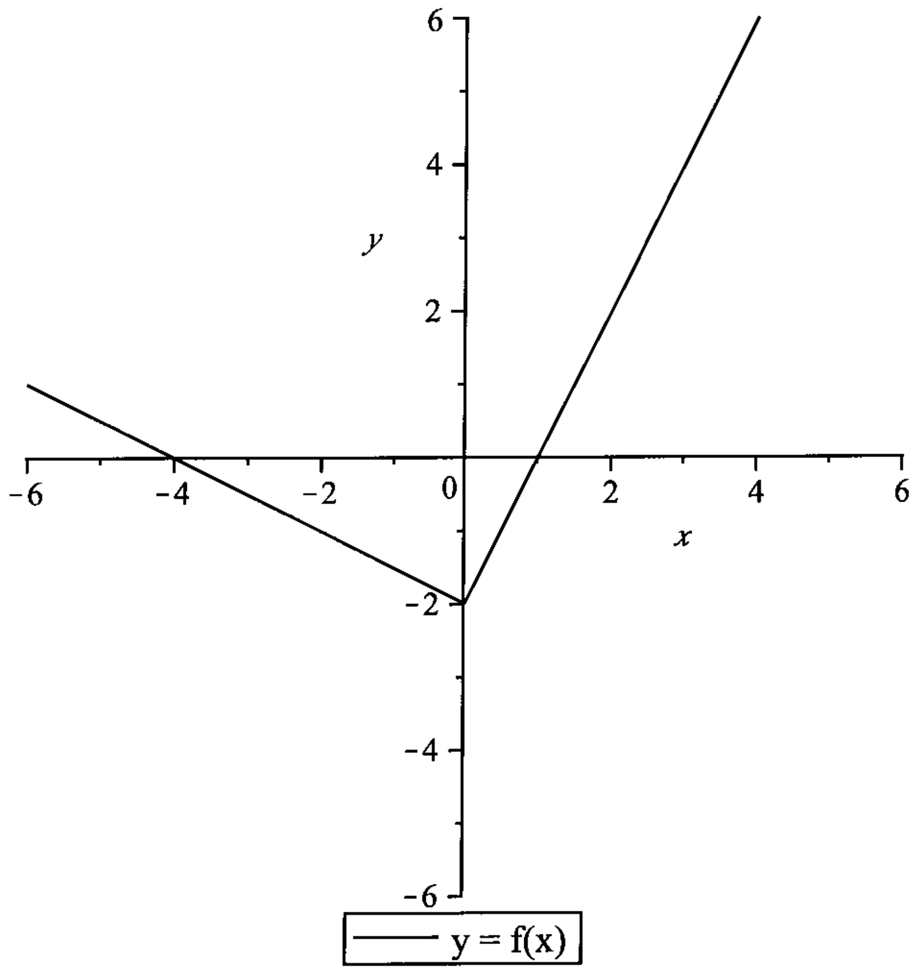


FIGURE 1. A function $y = f(x)$.

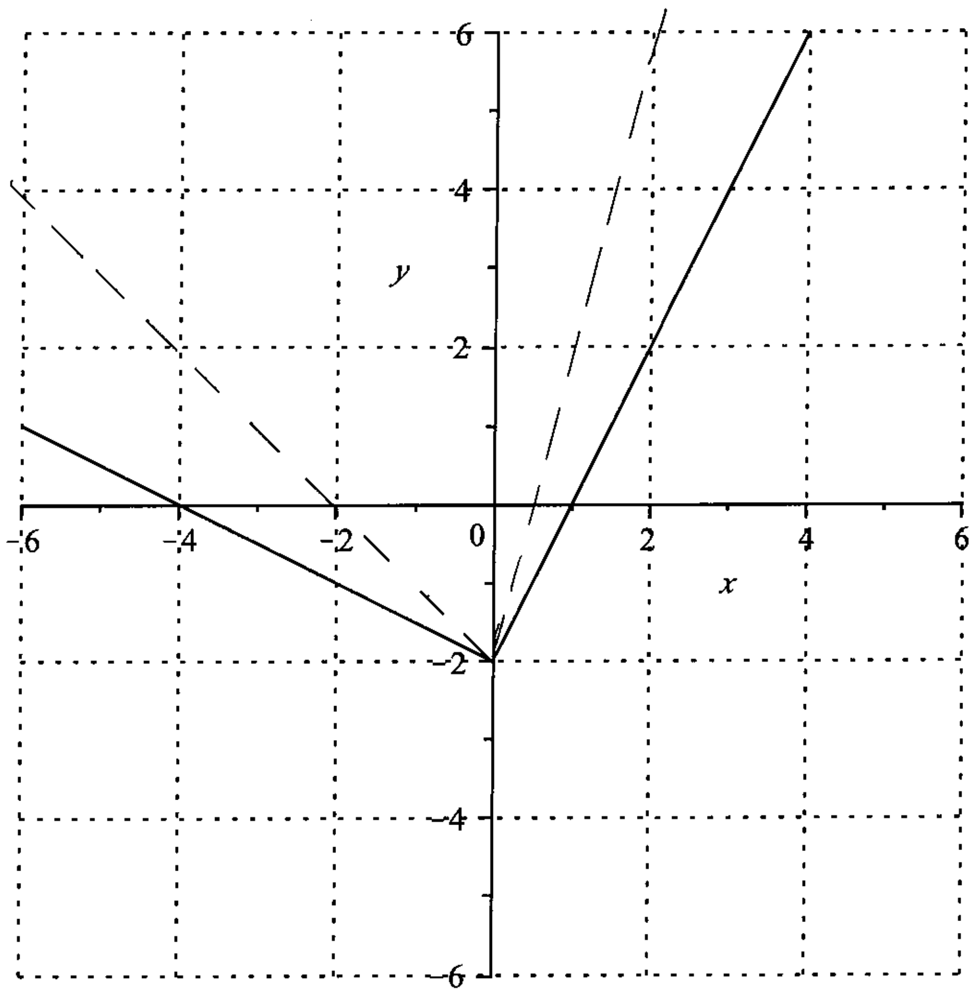


FIGURE 2. (a) $y = f(2x)$.

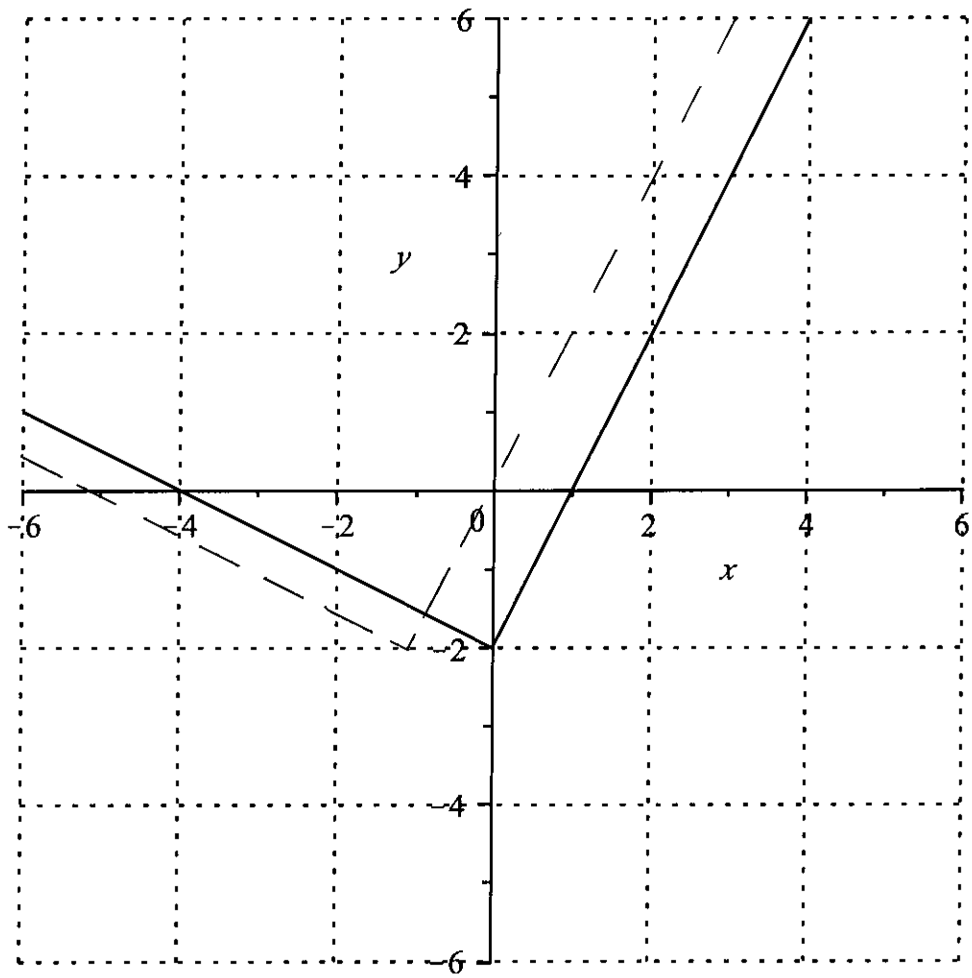


FIGURE 3. (b) $y = f(x + 1)$.

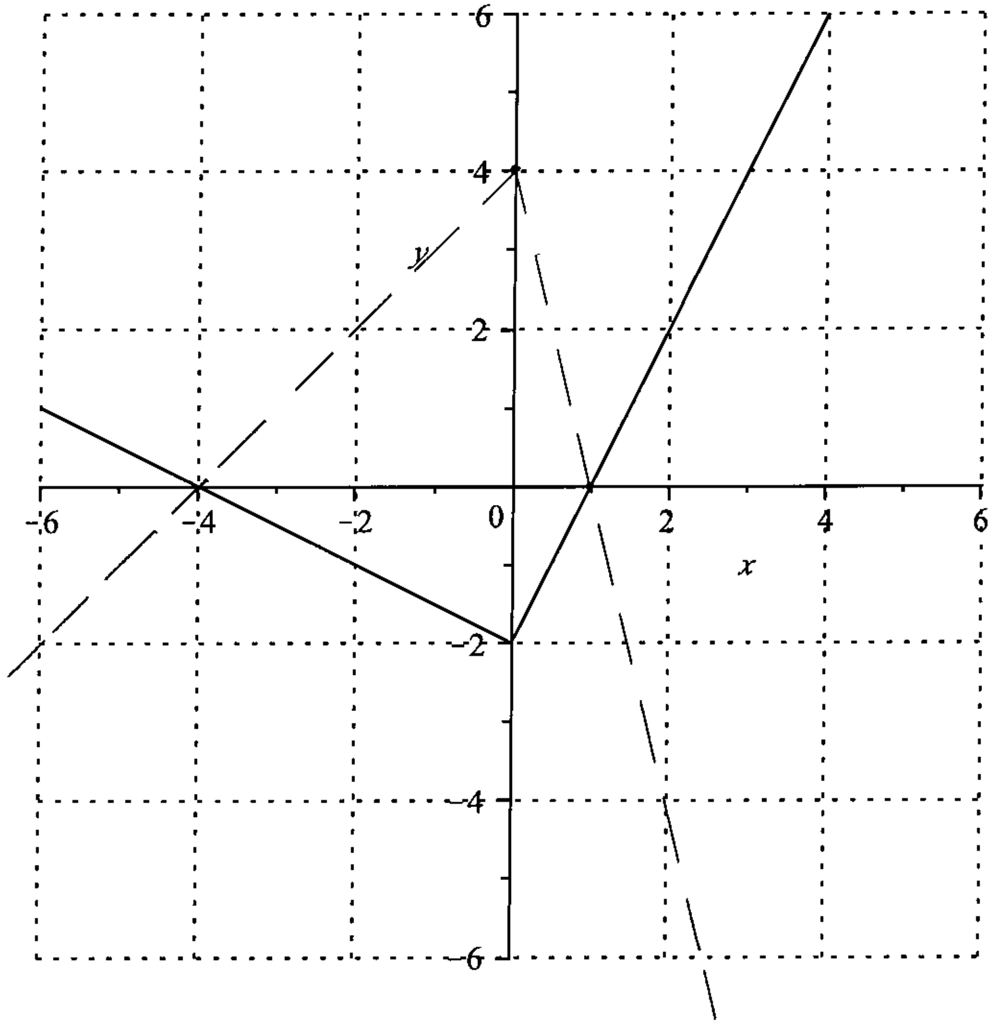


FIGURE 4. (d) $y = -2f(x)$.