CALCULUS A, TEST 3

This test is to be done working alone, and to be returned to the Library within 3 hours of the time you initially check it out from the Library.

You may use a calculator if you wish. You're not welcome to computers. In particular, don't just use *Sage* and believe the results it gives you. Show me how to get your results.

You need not simplify the results of your calculations. As always in math, you should show me what you are doing, explaining so a reader can understand and follow your work. When I ask for numerical answers, please give me exact expressions like e^π, not numerical approximations like 23.140692632779269007.

Please remember that you are writing to be read.

Do any 4 of the following 6 problems. Tell me which ones you want me to grade.

1. Compute

$$\int_0^1 (2x^2 + 5)\, dx$$

by writing a Riemann sum, evaluating that sum, and taking its limit. If it helps, you may use any of the formulas

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2},$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+\frac{1}{2})(n+1)}{3}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Remark: I really do want you to do this using Riemann sums, though obviously you could check your result by evaluating the integral using the Fundamental Theorem.

2. Let the function f and its derivatives be given by

$$f(x) = x + \frac{1}{x} + \frac{3}{x^2} = \frac{x^3 + x + 3}{x^2}$$

$$f'(x) = 1 - \frac{1}{x^2} - \frac{6}{x^3} = \frac{(x-2)(x^2+2x+3)}{x^3}$$

$$f''(x) = \frac{2}{x^3} + \frac{18}{x^4} = \frac{2x+18}{x^4}.$$

(a) Where is f increasing? Decreasing? What x values are local maxima? Minima?
(b) Where is f concave up? Down? What x values are inflection points?
(c) To what very simple function is $f(x)$ asymptotic when $|x|$ gets large?
(d) Give a rough sketch of $y = f(x)$, showing important features you identified in the previous parts of the problem and showing anything else you think is important.

3. Let $f(x) = x^5 - 5x^3 + 1$.

(a) Compute $f'(x)$ and $f''(x)$. Be sure you get this part right, or the rest of the problem will be completely hosed.

(b) Where is f increasing? Decreasing? What x values are local maxima? Minima?

(c) Where is f concave up? Down? What x values are inflection points?

(d) Sketch $y = f(x)$, showing important features you identified in the previous parts of the problem.

Note: As stated in the instructions, I would like exact values for the positions of extrema and inflection points, not numerical approximations. You will therefore actually have to do some calculus and algebra here.

4. An aquarium is a glass-sided container for live fish. Suppose you want to make an aquarium with a square base and with no top. (So it is shaped like a square-bottomed box with no lid.) You have enough money to buy 1 square meter of glass, which you will use to make both the bottom and the sides of the aquarium. Show how to use calculus to find the aquarium of largest volume you can afford to build.

5. Find exactly the global maximum and minimum for the function $\ln(2x^2 - 7x + 10)$ on the interval $0 \leq x \leq 3$.

What would happen if instead we looked at $\ln(2x^2 - 7x + 6)$?

6. Compute the following quantities, perhaps using the Fundamental Theorem of Calculus. For some of these, you just want to do the calculation. For at least one, though, you want to think instead of calculating.

(a) $\displaystyle\int_1^2 \left(x^2 + \frac{1}{x^2}\right) dx.$

(b) $\displaystyle\int_{\pi/4}^{\pi/2} \cos x \, dx.$

(c) $\displaystyle\int_{-1}^{1} \tan x \, dx.$

(d) $\displaystyle\frac{d}{dx}\int_1^x \sin(t^2)\, dt.$

(e) $\displaystyle\frac{d}{dx}\int_1^{x^2} \sin(t^2)\, dt.$