Cognitive Complexity of Phonological Patterns
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http://cs.earlham.edu/~jrogers/slides/UDel.ho.pdf

Joint work with Jeff Heinz, U. Delaware,
Geoff Pullum and Barbara Scholz, U.Edinburgh,
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Portions of this work completed while in residence at the
Radcliffe Institute for Advanced Study

Yawelmani Yokuts (Kissberth’73)

\[ \Sigma^{*}CCC\Sigma^{*} \]

Contrast: \[ \star C^{2i+1} \]

**Definition 1** A finite-state stringset is *one in which there is an a priori bound, independent of the length of the string, on the amount of information that must be inferred in distinguishing strings in the set from those not in the set.*

Regular = Recognizable = Finite-State
Cognitive Complexity of Simple Patterns

Sequences of ‘A’s and ‘B’s which end in ‘B’:
\[ S_0 \rightarrow AS_0, S_0 \rightarrow BS_0, S_0 \rightarrow B \]

\( (A + B)^*B \)

Sequences of ‘A’s and ‘B’s which contain an odd number of ‘B’s:
\[ S_0 \rightarrow AS_0, S_0 \rightarrow BS_1, \\
S_1 \rightarrow AS_1, S_1 \rightarrow BS_0, S_1 \rightarrow \varepsilon \]

\( (A^*BA^*BA^*)^*A^*BA^* \)

Some More Simple Patterns

Sequences of ‘A’s and ‘B’s which contain at least one ‘B’:
\[ S_0 \rightarrow AS_0, S_0 \rightarrow BS_1, \\
S_1 \rightarrow AS_1, S_1 \rightarrow BS_0, S_1 \rightarrow \varepsilon \]

\( A^*B(A + B)^* \)

Sequences of ‘A’s and ‘B’s which contain exactly one ‘B’:
\[ S_0 \rightarrow AS_0, S_0 \rightarrow BS_0, \\
S_1 \rightarrow AS_1, S_1 \rightarrow \varepsilon \]

\( A^*BA^* \)
Dual characterizations of complexity classes

Computational classes
• Characterized by abstract computational mechanisms
• Equivalence between mechanisms
• Tools to determine structural properties of stringsets

Descriptive classes
• Characterized by the nature of information about the properties of strings that determine membership
• Independent of mechanisms for recognition
• Subsume wide range of types of patterns

Cognitive Complexity from First Principles

What kinds of distinctions does a cognitive mechanism need to be sensitive to in order to classify an event with respect to a pattern?

Reasoning about patterns
• What objects/entities/things are we reasoning about?
• What relationships between them are we reasoning with?
Some Assumptions about Linguistic Behaviors

- Perceive/process/generate linear sequence of (sub)events
- Can model as strings—linear sequence of abstract symbols
  - Discrete linear order (initial segment of $\mathbb{N}$).
  - Labeled with alphabet of events
    Partitioned into subsets, each the set of positions at which some event occurs.

Word models

\[
\langle D, \preceq, \preceq^+, P_\sigma \rangle_{\sigma \in \Sigma}
\]

\[
\begin{aligned}
\langle D, \preceq, P_\sigma \rangle_{\sigma \in \Sigma} & \quad \langle \rangle \\
\langle D, \preceq^+, P_\sigma \rangle_{\sigma \in \Sigma} & \quad \langle +1 \rangle \\
\end{aligned}
\]

\[
D \quad \text{Finite}
\]

\[
\preceq^+ \quad \text{Linear order on } D
\]

\[
\preceq \quad \text{Successor wrt } \preceq^+
\]

\[
P_\sigma \quad \text{Subset of } D \text{ at which } \sigma \text{ occurs}
\]

\[
(P_\sigma \text{ partition } D)
\]

\[
CCVC = \langle \{0, 1, 2, 3\}, \{i, i+1\} \mid 0 \leq i < 3\rangle, \{0, 1, 3\}_C, \{2\}_V \rangle
\]

\[
\langle D \quad \preceq \quad P_C \quad P_V \rangle
\]
Adjacency—Substrings

\[
CVVCVC
\]

**Definition 2 (k-Factor)**

\(v\) is a factor of \(w\) if \(w = uvx\) for some \(u, v \in \Sigma^*\).

\(v\) is a \(k\)-factor of \(w\) if it is a factor of \(w\) and \(|v| = k\).

\[
F_k(w) \overset{\text{def}}{=} \begin{cases} 
\{v \in \Sigma^k \mid (\exists u, x \in \Sigma^*)[w = uvx]\} & \text{if } |w| \geq k, \\
\{w\} & \text{otherwise.}
\end{cases}
\]

\[F_2(CVCVCV) = \{CV, VC\}\]
\[F_7(CVCVCV) = \{CVCVCV\}\]

Strictly Local Stringsets—SL

Strictly \(k\)-Local Definitions

—Grammar is set of permissible \(k\)-factors

\[\mathcal{G} \subseteq F_k(\{\times\} \cdot \Sigma^* \cdot \{\times\})\]

\[w \models \mathcal{G} \overset{\text{def}}{\iff} F_k(\times \cdot w \cdot \times) \subseteq \mathcal{G}\]

\[L(\mathcal{G}) \overset{\text{def}}{=} \{w \mid w \models \mathcal{G}\}\]

e.g.:

\[\mathcal{G} = \{\times C, CV, VC, C\times\}, \quad L(\mathcal{G}) = CV(CV)^*C\]

**Definition 3 (Strictly Local Sets)** A stringset \(L\) over \(\Sigma\) is Strictly Local if there is some strictly \(k\)-local definition \(\mathcal{G}\) over \(\Sigma\) (for some \(k\)) such that \(L\) is the set of all strings that satisfy \(\mathcal{G}\)
SL Hierarchy

Definition 4 (SL)
A stringset is Strictly $k$-Local if it is definable with an SL$_k$ definition.

A stringset is Strictly Local (in SL) if it is SL$_k$ for some $k$.

Theorem 1 (SL-Hierarchy)

\[ SL_2 \subseteq SL_3 \subseteq \cdots \subseteq SL_i \subseteq SL_{i+1} \subseteq \cdots \subseteq SL \]

Every Finite stringset is SL$_k$ for some $k$: Fin $\subseteq$ SL.
There is no $k$ for which SL$_k$ includes all Finite languages.

$\star$ CCC is SL$_3$

\[ \mathcal{G}_{-CCC} = F_3(\{\times\} \cdot \Sigma^* \cdot \{\times\}) - \{CCC\} \]

Membership in an SL$_k$ stringset depends only on the individual $k$-factors which occur in the string.
Scanners

Recognizing an $SL_k$ string set requires only remembering the $k$ most recently encountered symbols.

Scanners as FSA

\[ A \overset{\text{def}}{=} (Q, \Sigma, q_0, \delta, F) \]
\[ Q \overset{\text{def}}{=} F_{k-1}(\Sigma^*) \cup \{\ast\} \cdot \bigcup_{0 \leq i < k-1} \{ F_{i<k-1}(\Sigma^*) \} \]
\[ q_0 \overset{\text{def}}{=} \ast \]
\[ \delta(\sigma \cdot v, \gamma) \overset{\text{def}}{=} u \iff u = v \cdot \gamma \in Q \lor u = \sigma \cdot v \cdot \gamma = \ast \cdot v \cdot \gamma \in Q \]
\[ F \overset{\text{def}}{=} Q \]
Character of Strictly $k$-Local Sets

**Theorem (Suffix Substitution Closure):**
A stringset $L$ is strictly $k$-local iff whenever there is a string $x$ of length $k - 1$ and strings $w$, $y$, $v$, and $z$, such that
\[
\begin{align*}
    w \cdot x \cdot y &\in L \\
v \cdot x \cdot z &\in L
\end{align*}
\]
then it will also be the case that
\[
w \cdot x \cdot z \in L
\]

E.g.: But $\star CCC$ is not $SL_2$:
\[
\begin{array}{ccc}
V \cdot VC \cdot CV & \in \star CCC & C \cdot C \cdot VC \in \star CCC \\
C \cdot VC \cdot VC & \in \star CCC & V \cdot C \cdot CV \in \star CCC \\
V \cdot VC \cdot VC & \in \star CCC & C \cdot C \cdot CV \notin \star CCC
\end{array}
\]

Cognitive interpretation of SL

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) $SL_k$ language must be sensitive, at least, to the length $k$ blocks of consecutive events that occur in the presentation of the string.

- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the immediately prior sequence of $k - 1$ events.

- Any cognitive mechanism that is sensitive only to the length $k$ blocks of consecutive events in the presentation of a string will be able to recognise only $SL_k$ languages.
Cambodian

- In words of all sizes, primary stress falls on the final syllable.
- In words of all sizes, secondary stress falls on all heavy syllables.
- Light syllables occur only immediately following heavy syllables.
- Light monosyllables do not occur.
Cambodian—Primary stress falls on the final syllable

Cambodian—Light syllables occur only immediately following heavy syllables
Cambodian—Minimized

Alawa

\[ G_{\text{Alawa}} = \{ \times \sigma, \times \sigma \sigma, \times \sigma \kappa, \sigma \sigma, \sigma \sigma \sigma, \sigma \sigma \sigma \sigma, \sigma \sigma \sigma \kappa, \sigma \sigma \sigma \} \]
Alawa

\[ \begin{align*}
\mathcal{G}_{\text{ArabicBH}} &= \{\ldots\} - \{\sigma\delta_0\kappa \mid \sigma \in \sigma_0, \sigma_1, \sigma_2\} \\
L_{\text{ArabicBH}} &= L(\ldots) \cap L_{\sigma_0}\sigma_1\kappa
\end{align*} \]
Strictly Local Stress Patterns

Heinz’s Stress Pattern Database (ca. 2007)—109 patterns

- 9 are SL_2: Abun West, Afrikans, … Cambodian, … Maranungku
- 44 are SL_3: Alawa, Arabic (Bani-Hassan), …
- 24 are SL_4: Arabic (Cairene), …
- 3 are SL_5: Asheninca, Bhojpuri, Hindi (Fairbanks)
- 1 is SL_6: Icua Tupi
- 28 are not SL: Amele, Bhojpuri (Shukla Tiwari), Arabic Classical, Hindi (Keldar), Yidin, …

72% are SL, all $k \leq 6$. 49% are SL_3.
The Problematic Case—Some-$\sigma$

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Locally definable stringsets

$$f \in F_k(\times \cdot \Sigma^* \cdot \land \cdot \lor \cdot \neg) \quad w \models f \iff f \in F_k(\times \cdot w \cdot \lor \cdot \neg)$$

$$\varphi \land \psi \quad w \models \varphi \land \psi \iff w \models \varphi \land w \models \psi$$

$$\neg \varphi \quad w \models \neg \varphi \iff w \not\models \varphi$$

Definition 5 (Locally Testable Sets) A stringset $L$ over $\Sigma$ is Locally Testable iff (by definition) there is some $k$-expression $\varphi$ over $\Sigma$ (for some $k$) such that $L$ is the set of all strings that satisfy $\varphi$: $L = L(\varphi) \overset{\text{def}}{=} \{ w \in \Sigma^* \mid w \models \varphi \}$

$$SL_k \equiv \bigwedge_{f_i \not\in G} [\neg f_i] \subseteq LT_k$$
Some-\(\ddot{\sigma}\)

\[
\varphi_{\text{Some-}\ddot{\sigma}} = (\times \ddot{\sigma} \lor \ddot{\sigma} \kappa) \land \ddot{\sigma} \quad \text{Starts or ends with } \ddot{\sigma}
\]

LT Automata

Membership in an LT\(_k\) stringset depends only on the set of \(k\)-Factors which occur in the string.

Recognizing an LT\(_k\) stringset requires only remembering which \(k\)-factors occur in the string.
Character of Locally Testable sets

**Theorem 2 (k-Test Invariance)** A stringset $L$ is Locally Testable iff there is some $k$ such that, for all strings $x$ and $y$,

if $\preceq x \preceq$ and $\preceq y \preceq$ have exactly the same set of $k$-factors then either both $x$ and $y$ are members of $L$ or neither is.

**Definition 6 (k-Local Equivalence)**

$$w \equiv_k v \overset{\text{def}}{\iff} F_k(\preceq w \preceq) = F_k(\preceq v \preceq).$$

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LT Hierarchy

**Definition 7 (LT)**

A stringset is $k$-Locally Testable if it is definable with an $LT_k$-expression.

A stringset is Locally Testable (in LT) if it is $LT_k$ for some $k$.

**Theorem 3 (LT-Hierarchy)**

$$LT_2 \subsetneq LT_3 \subsetneq \cdots \subsetneq LT_1 \subsetneq LT_{i+1} \subsetneq \cdots \subsetneq LT.$$
Cognitive interpretation of LT

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) LT$_k$ language must be sensitive, at least, to the set of length $k$ contiguous blocks of events that occur in the presentation of the string—both those that do occur and those that do not.

- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the set of length $k$ blocks of events that occurred at any prior point.

- Any cognitive mechanism that is sensitive *only* to the occurrence or non-occurrence of length $k$ contiguous blocks of events in the presentation of a string will be able to recognise *only* LT$_k$ languages.
**FO(+1)**

Models: \( (D, \prec, P_\sigma)_{\sigma \in \Sigma} \)

First-order Quantification (over positions in the strings)

\[
x \prec y \quad w, [x \mapsto i, y \mapsto j] \models x \prec y \quad \iff \quad j = i + 1
\]

\[
P_\sigma(x) \quad w, [x \mapsto i] \models P_\sigma(x) \quad \iff \quad i \in P_\sigma
\]

\[
\varphi \wedge \psi
\]

\[
\neg \varphi
\]

\[
(\exists x)[\varphi(x)]\quad w, s \models (\exists x)[\varphi(x)] \quad \iff \quad w, s[x \mapsto i] \models \varphi(x)
\]

for some \( i \in D \)

**FO(+1)-Definable Stringsets:**

\[
L(\varphi) \overset{\text{def}}{=} \{ w \mid w \models \varphi \}.
\]

One-\( \sigma = L((\exists x)[\sigma(x) \wedge (\forall y)[\sigma(y) \rightarrow x \approx y]]) \)

Arabic (Classical) is FO(+1)

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**Character of the FO(+1) Definable Stringsets**

**Definition 8 (Locally Threshold Testable)** A set \( L \) is Locally Threshold Testable (LTT) iff there is some \( k \) and \( t \) such that, for all \( w, v \in \Sigma^* \):

- if for all \( f \in F_k(\ast \cdot w \cdot \ast) \cup F_k(\ast \cdot v \cdot \ast) \)
  - either \( |w|_f = |v|_f \) or both \( |w|_f \geq t \) and \( |v|_f \geq t \),

then \( w \in L \iff v \in L \).

**Theorem 4 (Thomas)** A set of strings is First-order definable over \( (D, \prec, P_\sigma)_{\sigma \in \Sigma} \) iff it is Locally Threshold Testable.

Membership in an FO(+1) definable stringset depends only on the multiplicity of the \( k \)-factors, up to some fixed finite threshold, which occur in the string.
LTT Automata

Cognitive interpretation of FO(+1)

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) FO(+1) stringset must be sensitive, at least, to the multiplicity of the length \( k \) blocks of events, for some fixed \( k \), that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold \( t \).

- If the strings are presented as sequences of events in time, then this corresponds to being able count up to some fixed threshold.

- Any cognitive mechanism that is sensitive only to the multiplicity, up to some fixed threshold, (and, in particular, not to the order) of the length \( k \) blocks of events in the presentation of a string will be able to recognize only FO(+1) stringsets.
No $H$ before $\dot{H}$ is not FO(+1)

Primary stress on leftmost heavy syllable

$$\ast \, H \ldots \dot{H}$$

First-Order($<$) definable stringsets

$$\langle D, \lessdot^+, P_\sigma \rangle_{\sigma \in \Sigma}$$

First-order Quantification over positions in the strings

\begin{align*}
x \lessdot^+ y & \quad w, [x \mapsto i, y \mapsto j] \models x \lessdot^+ y \quad \text{def} \quad i < j \\
P_\sigma(x) & \quad w, [x \mapsto i] \models P_\sigma(x) \quad \text{def} \quad i \in P_\sigma \\
\varphi \land \psi & \quad : \\
\neg \varphi & \quad : \\
(\exists x)[\varphi(x)] & \quad w, s \models (\exists x)[\varphi(x)] \quad \text{def} \quad w, s[x \mapsto i] \models \varphi(x) \\
& \quad \text{for some } i \in D
\end{align*}
Star-Free stringsets

**Definition 9 (Star-Free Set)** The class of Star-Free Sets (SF) is the smallest class of languages satisfying:

- $\text{Fin} \subseteq \text{SF}$.

- If $L_1, L_2 \in \text{SF}$ then:
  - $L_1 \cdot L_2 \in \text{SF}$,
  - $L_1 \cup L_2 \in \text{SF}$,
  - $\overline{L_1} \in \text{SF}$.

**Theorem 5 (McNauthton and Papert)** A set of strings is **First-order definable over** $\langle D, \preceq, P_\sigma \rangle_{\sigma \in \Sigma}$ **iff** it is Star-Free.

Cognitive interpretation of SF (FO($<$))

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) SF language must be sensitive, at least, to both the order and the multiplicity of the length $k$ blocks of events, for some fixed $k$, that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold $t$.

- If the strings are presented as sequences of events in time, then this corresponds to being able not only to count events up to some threshold but also to track the sequence in which those events occur.

- Any cognitive mechanism that is sensitive *only* to the order and the multiplicity of the length $k$ blocks of events, for some fixed $k$, that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold $t$ will be able to recognise *only* SF languages.
Sub-regular Hierarchies

Yidin
- Primary stress on the leftmost heavy syllable, else the initial syllable
- Secondary stress iteratively on every second syllable in both directions from primary stress
- No light monosyllables

Explicitly:
- Exactly one $\hat{\sigma}$ (One-$\hat{\sigma}$)
- $\hat{L}$ implies no $H$
  (No-$H$-with-$\hat{L}$)
- $\sigma$ and $\hat{\sigma}/\sigma$ alternate
  (Alt)
- First $H$ gets primary stress
  (No-$H$-before-$\hat{H}$)
- $\hat{L}$ only if initial
  (Nothing-before-$\hat{L}$)
- No $\hat{L}$ monosyllables
  (No $\times\hat{L}\times$)
Classifying Conjunctive Constraints

- One-\(\sigma\) \(\exists x [\sigma(x)]\) (LTT\(_{1,2}\))
- No-\(H\)-before-\(\bar{H}\) \(\neg (\exists x, y) [x \sigma^+ y \land H(x) \land \bar{H}(y)]\) (SF)
- No-\(H\)-with-\(\bar{L}\) \(\neg (H \land \bar{L})\) (LT\(_1\))
- Nothing-before-\(\bar{L}\) \(\neg \sigma L\) (SL\(_2\))
- Alt \(\neg \sigma \sigma \land \neg \sigma \sigma \land \neg \sigma \sigma \land \neg \sigma \sigma \land \neg \sigma \sigma\) (SL\(_3\))
- No \(\times \bar{L} \times\) \(\neg \times \bar{L} \times\) (SL\(_4\))

Yidin is SF

Combining Conjunctive Constraints

- One-\(\sigma\)

- No-\(H\)-before-\(\bar{H}\)

- One-\(\sigma\) \(\cap\) No-\(H\)-before-\(\bar{H}\)
• No-$H$-with-$\hat{L}$

• Nothing-before-$\hat{L}$

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• Alt

• No $\times \hat{L} \times$

Yidin

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Precedence—Subsequences

Definition 10 (Subsequences)

\[ v \sqsubseteq w \iff v = \sigma_1 \cdots \sigma_n \text{ and } w \in \Sigma^* \cdot \Sigma^* \cdots \Sigma^* \cdot \Sigma^* \cdot \Sigma^* \]

\[ P_k(w) \overset{\text{def}}{=} \{ v \in \Sigma^k \mid v \sqsubseteq w \} \]

\[ P_{\leq k}(w) \overset{\text{def}}{=} \{ v \in \Sigma^{\leq k} \mid v \sqsubseteq w \} \]

\[ P_2(\sigma\sigma\sigma\sigma\sigma) = \{ \sigma\sigma, \sigma\sigma, \sigma\sigma, \sigma\sigma, \sigma\sigma \} \]

\[ P_{\leq 2}(\sigma\sigma\sigma\sigma\sigma) = \{ \varepsilon, \sigma, \sigma, \sigma, \sigma, \sigma, \sigma, \sigma, \sigma, \sigma, \sigma \} \]

Strictly Piecewise Stringsets—SP

Strictly \( k \)-Piecewise Definitions

\[ G \subseteq \Sigma^{\leq k} \]

\[ w \models G \overset{\text{def}}{=} P_{\leq k}(w) \subseteq P_k(G) \]

\[ L(G) \overset{\text{def}}{=} \{ w \in \Sigma^* \mid w \models G \} \]

\[ G_{\text{No-H-before-H}} = \{ HH, H\hat{H}, \hat{H}H, \hat{H}\hat{H}, \hat{H}H, \hat{H}\hat{H}, \ldots \} \]

Membership in an SP\(_k\) stringset depends only on the individual \((\leq k)\)-subsequences which do and do not occur in the string.
Character of the Strictly $k$-Piecewise Sets

**Theorem 6** A stringset $L$ is Strictly $k$-Piecewise Testable iff it is closed under subsequence:

$$wσv ∈ L ⇒ wv ∈ L$$

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Every naturally occurring stress pattern requires Primary Stress

⇒

No naturally occurring stress pattern is SP.

But SP can forbid multiple primary stress: $¬σσ$

---

**Yidin constraints wrt SP**

- One-$σ$ is not SP
  - $σσ ⊆ σσσ$
- No-$H$-before-$H$ is SP$_2$
  - $¬H˙H$
- No-$H$-with-$L$ is SP$_2$
  - $¬H˙L ∧ ¬LH$
- Nothing-before-$L$ is SP$_2$
  - $¬σL$
- Alt is not SP
  - $σσσ ⊆ σσσσ$
- No $∗L$ is not SP
  - $L ⊆ L L$
Cognitive interpretation of SP

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) SP\(_k\) stringset must be sensitive, at least, to the length \(k\) (not necessarily consecutive) sequences of events that occur in the presentation of the string.

- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to up to \(k - 1\) events distributed arbitrarily among the prior events.

- Any cognitive mechanism that is sensitive only to the length \(k\) sequences of events in the presentation of a string will be able to recognize only SP\(_k\) stringsets.

\(k\)-Piecewise Testable Stringsets

PT\(_k\)-expressions

\[
\begin{align*}
p \in \Sigma^{\leq k} & \quad \implies \quad w \models p \iff p \sqsubseteq w \\
\varphi \land \psi & \quad \implies \quad w \models \varphi \land \psi \iff w \models \varphi \text{ and } w \models \psi \\
\neg \varphi & \quad \implies \quad w \models \neg \varphi \iff w \not\models \varphi
\end{align*}
\]

\(k\)-Piecewise Testable Languages (PT\(_k\)):

\[
L(\varphi) \overset{\text{def}}{=} \{w \in \Sigma^* \mid w \models \varphi\}
\]

One-\(\delta = L(\delta \land \neg \delta \delta)\)

Membership in an PT\(_k\) stringset depends only on the set of \((\leq k)\)-subsequences which occur in the string.

SP\(_k\) is equivalent to \(\bigwedge_{p_i \not\models \varphi} \neg p_i\)
Character of Piecewise Testable sets

**Theorem 7 (k-Subsequence Invariance)** A stringset $L$ is Piecewise Testable iff

there is some $k$ such that, for all strings $x$ and $y$,

if $x$ and $y$ have exactly the same set of $(\leq k)$-subsequences

then either both $x$ and $y$ are members of $L$ or neither is.

\[ w \equiv_k^P v \overset{\text{def}}{\iff} P_{\leq k}(w) = P_{\leq k}(v). \]

---

**Yidin constraints wrt PT**

- One-$\hat{\sigma}$ is PT$_2$
  \[ \hat{\sigma} \land \neg \hat{\sigma} \hat{\sigma} \]

- No-$H$-before-$\hat{H}$ is SP$_2$
  \[ \neg H \hat{H} \]

- No-$H$-with-$\hat{L}$ is SP$_2$
  \[ \neg H \hat{L} \land \neg \hat{L} H \]

- Nothing-before-$\hat{L}$ is SP$_2$
  \[ \neg \sigma \hat{L} \]

- Alt is not PT
  \[ \hat{\sigma} \hat{\sigma} \cdots \hat{\sigma} \hat{\sigma} \equiv \hat{\sigma}^{2k} \]

- No $\hat{\sigma} \hat{\sigma} \cdots \hat{\sigma} \hat{\sigma} \hat{\sigma}$ is PT$_2$
  \[ \hat{L} \rightarrow (\hat{\sigma} \hat{L} \lor \hat{L} \sigma) \]
Cognitive interpretation of PT

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) PT\(_k\) stringset must be sensitive, at least, to the set of length \(k\) subsequences of events that occur in the presentation of the string—both those that do occur and those that do not.

- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the set of all length \(k\) subsequences of the sequence of prior events.

- Any cognitive mechanism that is sensitive only to the set of length \(k\) subsequences of events in the presentation of a string will be able to recognize only PT\(_k\) stringsets.

Yidin wrt Local and Piecewise Constraints

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<table>
<thead>
<tr>
<th>Constraints</th>
<th>Sense 1</th>
<th>Sense 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-(\sigma)</td>
<td>LTT(_{1,2})</td>
<td>PT(_2)</td>
</tr>
<tr>
<td>Some-(\sigma)</td>
<td>LT(_1)</td>
<td>PT(_1)</td>
</tr>
<tr>
<td>At-Most-One-(\sigma)</td>
<td>LTT(_{1,2})</td>
<td>SP(_2)</td>
</tr>
<tr>
<td>No-(H)-before-(\tilde{H})</td>
<td>SF</td>
<td>SP(_2)</td>
</tr>
<tr>
<td>No-(H)-with-(\tilde{L})</td>
<td>LT(_1)</td>
<td>SP(_2)</td>
</tr>
<tr>
<td>Nothing-before-(\tilde{L})</td>
<td>SL(_2)</td>
<td>SP(_2)</td>
</tr>
<tr>
<td>Alt</td>
<td>SL(_2)</td>
<td>SF</td>
</tr>
<tr>
<td>No (\times\tilde{L}\times)</td>
<td>SL(_3)</td>
<td>PT(_2)</td>
</tr>
</tbody>
</table>

Yidin is co-occurrence of SL and PT constraints or of LT and SP constraints
Local and Piecewise Hierarchies

MSO definable stringsets

\[ \langle D, \preceq, \preceq^+, P_\sigma \rangle_{\sigma \in \Sigma} \]

First-order Quantification (positions)

Monadic Second-order Quantification (sets of positions)

\[ \preceq^+ \text{ is MSO-definable from } \preceq. \]
Character of the MSO-definable sets

Theorem 8 (Medvedev, Büchi, Elgot) A set of strings is MSO-definable over $\langle D, \triangleleft, \triangleleft^+, P_\sigma \rangle_{\sigma \in \Sigma}$ iff it is regular.

Theorem 9 (Chomsky Schützenberger) A set of strings is Regular iff it is a homomorphic image of a Strictly 2-Local set.

Theorem 10 $\text{MSO} = \exists \text{MSO}$ over strings.

Cognitive interpretation of Finite-state

- Any cognitive mechanism that can distinguish member strings from non-members of a finite-state stringset must be capable of classifying the events in the input into a finite set of abstract categories and are sensitive to the sequence of those categories.

- Subsumes any recognition mechanism in which the amount of information inferred or retained is limited by a fixed finite bound.

- Any cognitive mechanism that has a fixed finite bound on the amount of information inferred or retained in processing sequences of events will be able to recognize only finite-state stringsets.
Local and Piecewise Hierarchies

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