Cognitive Complexity of Phonological Patterns

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Slide 1

http://cs.earlham.edu/~jrogers/slides/UDel.ho.pdf

Joint work with Jeff Heinz, U. Delaware,

Geoff Pullum and Barbara Scholz, U.Edinburgh,

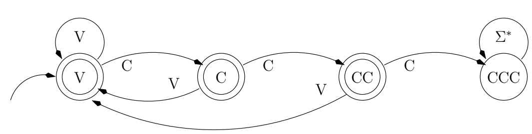
and a raft of Earlham College undergrads.

Portions of this work completed while in residence at the Radcliffe Institute for Advanced Study

Yawelmani Yokuts (Kissberth'73)

$$\star CCC$$

$$\overline{\Sigma^* CCC \Sigma^*}$$



Slide 2

Contrast: $\star C^{2i+1}$

Definition 1 *A* finite-state stringset *is one in which there is an* a priori *bound, independent of the length of the string, on the amount of information that must be inferred in distinguishing strings in the set from those not in the set.*

Regular = Recognizable = Finite-State

Cognitive Complexity of Simple Patterns

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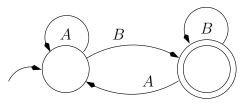
Sequences of 'A's and 'B's which end in 'B':

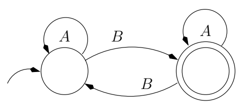
$$S_0 \longrightarrow AS_0,\ S_0 \longrightarrow BS_0,\ S_0 \longrightarrow B$$



$$(A+B)^*B$$

Sequences of 'A's and 'B's which contain an odd number of 'B's:

$$S_0 \longrightarrow AS_0,\ S_0 \longrightarrow BS_1,$$
$$S_1 \longrightarrow AS_1,\ S_1 \longrightarrow BS_0,\ S_1 \longrightarrow \varepsilon$$



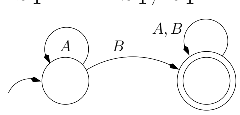
$$(A^*BA^*BA^*)^*A^*BA^*$$

Some More Simple Patterns

Slide 4

Sequences of 'A's and 'B's which contain at least one 'B':

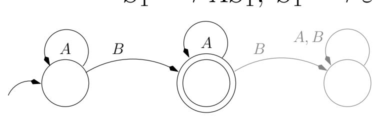
$$S_0 \longrightarrow AS_0,\ S_0 \longrightarrow BS_1,$$
$$S_1 \longrightarrow AS_1,\ S_1 \longrightarrow BS_1,\ S_1 \longrightarrow \varepsilon$$



$$A^*B(A+B)^*$$

Sequences of 'A's and 'B's which contain exactly one 'B':

$$S_0 \longrightarrow AS_0,\ S_0 \longrightarrow BS_1,$$
$$S_1 \longrightarrow AS_1,\ S_1 \longrightarrow \varepsilon$$



$$A^*BA^*$$

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Dual characterizations of complexity classes

Computational classes

- Characterized by abstract computational mechanisms
- Equivalence between mechanisms
- Tools to determine structural properties of stringsets

Descriptive classes

- Characterized by the nature of information about the properties of strings that determine membership
- Independent of mechanisms for recognition
- Subsume wide range of types of patterns

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Cognitive Complexity from First Principles

What kinds of distinctions does a cognitive mechanism need to be sensitive to in order to classify an event with respect to a pattern?

Reasoning about patterns

- What objects/entities/things are we reasoning about?
- What relationships between them are we reasoning with?

Some Assumptions about Linguistic Behaviors

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- Perceive/process/generate linear sequence of (sub)events
- Can model as strings—linear sequence of abstract symbols
  - Discrete linear order (initial segment of $\mathbb{N}$).
  - Labeled with alphabet of events

    Partitioned into subsets, each the set of positions at which some event occurs.

Word models

Slide 8

$$\langle \mathcal{D}, \triangleleft, \triangleleft^+, P_\sigma \rangle_{\sigma\in\Sigma}$$

$(+1)\quad \langle \mathcal{D}, \triangleleft, P_\sigma \rangle_{\sigma\in\Sigma} \qquad\qquad (<)\quad \langle \mathcal{D}, \triangleleft^+, P_\sigma \rangle_{\sigma\in\Sigma}$

$\mathcal{D}$	—	Finite
$\triangleleft^+$	—	Linear order on $\mathcal{D}$
$\triangleleft$	—	Successor wrt $\triangleleft^+$
P_σ	—	Subset of $\mathcal{D}$ at which σ occurs
		(P_σ partition $\mathcal{D}$)

$$\begin{array}{rlccccl} CCVC = & \langle & \{0,1,2,3\}, & \{\langle i, i+1\rangle \mid 0 \leq i < 3\}, & \{0,1,3\}_C, & \{2\}_V & \rangle \\ & \langle & \mathcal{D} & \triangleleft & P_C & P_V & \rangle \end{array}$$

Adjacency—Substrings

$$\overset{\frown}{C V C V C V}$$

Definition 2 (k-Factor)
v *is a* factor *of* w *if* $w = uvx$ *for some* $u, v \in \Sigma^*$.

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v *is a* k-factor *of* w *if it is a factor of* w *and* $|v| = k$.

$$F_k(w) \stackrel{def}{=} \begin{cases} \{v \in \Sigma^k \mid (\exists u, x \in \Sigma^*)[w = uvx]\} & \textit{if } |w| \geq k, \\ \{w\} & \textit{otherwise.} \end{cases}$$

$$F_2(CVCVCV) = \{CV,\ VC\}$$
$$F_7(CVCVCV) = \{CVCVCV\}$$

Strictly Local Stringsets—SL

Strictly k-Local Definitions
—Grammar is set of permissible k-factors

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$$\mathcal{G} \subseteq F_k(\{\rtimes\} \cdot \Sigma^* \cdot \{\ltimes\})$$
$$w \models \mathcal{G} \stackrel{\text{def}}{\Longleftrightarrow} F_k(\rtimes \cdot w \cdot \ltimes) \subseteq \mathcal{G}$$
$$L(\mathcal{G}) \stackrel{\text{def}}{=} \{w \mid w \models \mathcal{G}\}$$

e.g.:

$$\mathcal{G} = \{\rtimes C, CV, VC, C\ltimes\}, \qquad L(\mathcal{G}) = CV(CV)^*C$$

Definition 3 (Strictly Local Sets) *A stringset* L *over* Σ *is* Strictly Local *iff there is some strictly* k*-local definition* $\mathcal{G}$ *over* Σ *(for some* k*) such that* L *is the set of all strings that satisfy* $\mathcal{G}$

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SL Hierarchy

Definition 4 (*SL*)
A stringset is Strictly k-Local *if it is definable with an* SL_k *definition.*

A stringset is Strictly Local *(in SL) if it is* SL_k *for some* k*.*

Theorem 1 (SL-Hierarchy)

$$SL_2 \subsetneq SL_3 \subsetneq \cdots \subsetneq SL_i \subsetneq SL_{i+1} \subsetneq \cdots \subsetneq SL$$

Every Finite stringset is SL_k for some k: Fin $\subseteq$ SL.

There is no k for which SL_k includes all Finite languages.

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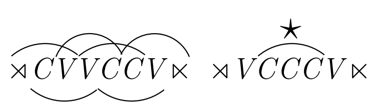
$\star$ *CCC* is SL_3

$$\mathcal{G}_{\neg CCC} = F_3(\{\rtimes\} \cdot \Sigma^* \cdot \{\ltimes\}) - \{CCC\}$$



Membership in an SL_k stringset depends only on the individual k-factors which occur in the string.

Scanners

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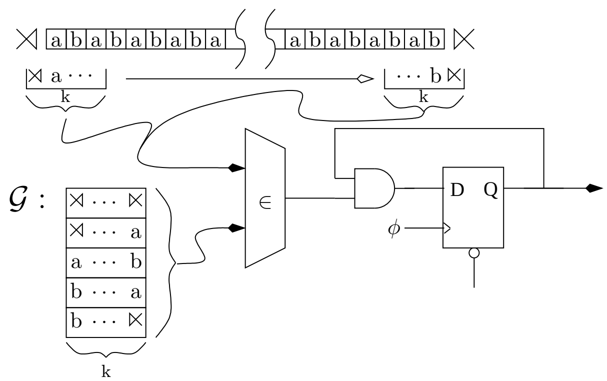

Recognizing an SL_k stringset requires only remembering the k most recently encountered symbols.

Scanners as FSA

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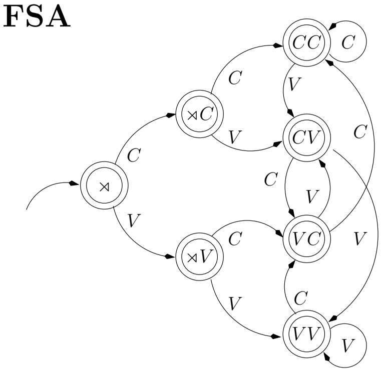

$$\mathcal{M} \stackrel{\text{def}}{=} \langle Q, \Sigma, q_0, \delta, F\rangle$$

$$\begin{aligned}
Q &\stackrel{\text{def}}{=} F_{k-1}(\Sigma^*) \cup \{\rtimes\} \cdot \bigcup\nolimits_{0 \le i < k-1}[\{F_{i<k-1}(\Sigma^*)\}] \\
q_o &\stackrel{\text{def}}{=} \rtimes \\
\delta(\sigma \cdot v, \gamma) &\stackrel{\text{def}}{=} u \Leftrightarrow u = v \cdot \gamma \in Q \;\vee\; u = \sigma \cdot v \cdot \gamma = \rtimes \cdot v \cdot \gamma \in Q \\
F &\stackrel{\text{def}}{=} Q
\end{aligned}$$

Character of Strictly k-Local Sets

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Theorem (Suffix Substitution Closure):
A stringset L is strictly k-local iff whenever there is a string x of length $k-1$ and strings w, y, v, and z, such that

$$\begin{array}{ccccccc} w & \cdot & \overbrace{x}^{k-1} & \cdot & y & \in L \\ v & \cdot & x & \cdot & z & \in L \end{array}$$

then it will also be the case that

$$w \cdot x \cdot z \in L$$

E.g.:

$$\begin{array}{ccccccc} V & \cdot & VC & \cdot & CV & \in \star CCC \\ C & \cdot & VC & \cdot & VC & \in \star CCC \\ \hline V & \cdot & VC & \cdot & VC & \in \star CCC \end{array}$$

But $\star CCC$ is not SL_2:

$$\begin{array}{ccccccc} C & \cdot & C & \cdot & VC & \in \star CCC \\ V & \cdot & C & \cdot & CV & \in \star CCC \\ \hline C & \cdot & C & \cdot & CV & \notin \star CCC \end{array}$$

Cognitive interpretation of SL

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- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) SL_k language must be sensitive, at least, to the length k blocks of consecutive events that occur in the presentation of the string.
- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the immediately prior sequence of $k-1$ events.
- Any cognitive mechanism that is sensitive *only* to the length k blocks of consecutive events in the presentation of a string will be able to recognise *only* SL_k languages.

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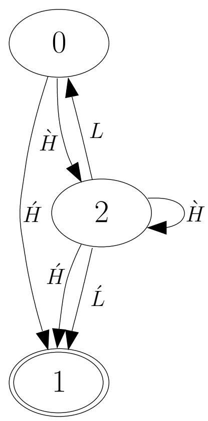
Cambodian



- In words of all sizes, primary stress falls on the final syllable.
- In words of all sizes, secondary stress falls on all heavy syllables.
- Light syllables occur only immediately following heavy syllables.
- Light monosyllables do not occur.

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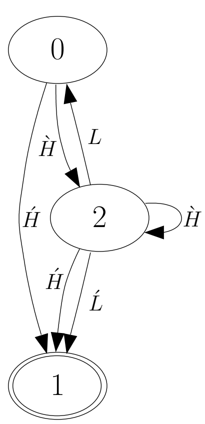
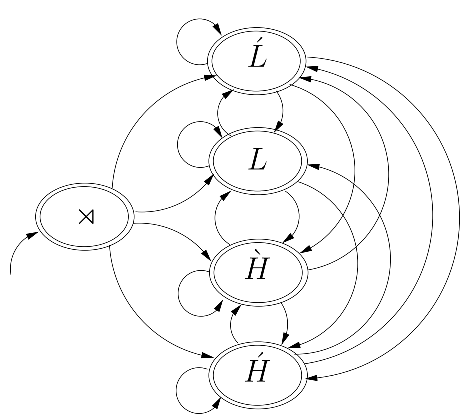
Cambodian





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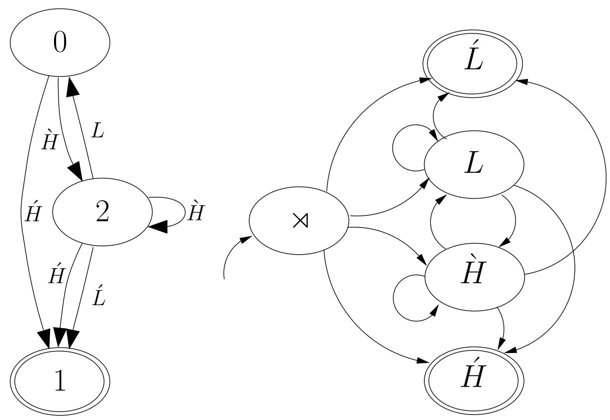
Cambodian—Primary stress falls on the final syllable



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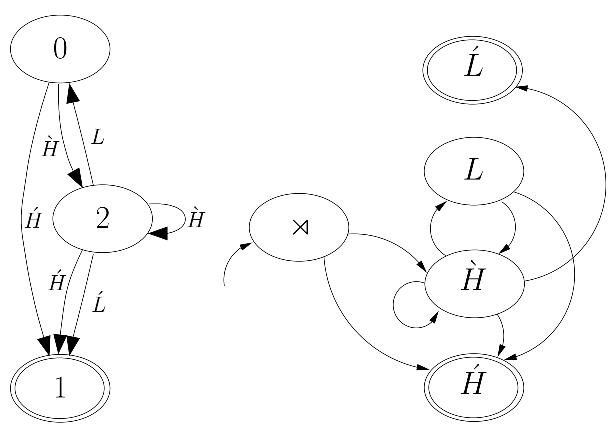
Cambodian—Light syllables occur only immediately following heavy syllables



Cambodian—Minimized

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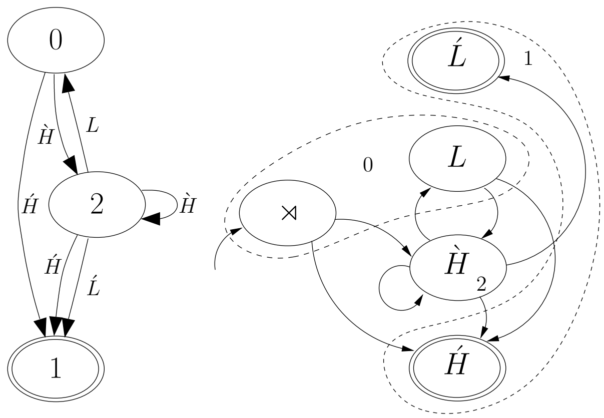

Alawa

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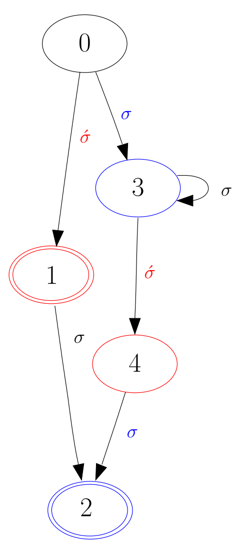

$\rtimes\sigma$	$\acute{\sigma}$	$\sigma\ltimes$
$\rtimes$	$\acute{\sigma}$	$\ltimes$
$\star\ \rtimes\sigma$	$\acute{\sigma}$	$\ltimes$

$\rtimes\sigma$	σ	$\acute{\sigma}\sigma\ltimes$
$\rtimes\acute{\sigma}$	σ	$\ltimes$
$\star\ \rtimes\sigma$	σ	$\ltimes$

$$\mathcal{G}_{\text{Alawa}} = \{\ \rtimes\sigma\sigma,\ \rtimes\sigma\acute{\sigma},\ \rtimes\acute{\sigma}\sigma,\ \sigma\sigma\sigma,\ \sigma\sigma\acute{\sigma},\ \sigma\acute{\sigma}\sigma,\ \acute{\sigma}\sigma\ltimes,\ \rtimes\acute{\sigma}\ltimes\ \}$$

Alawa

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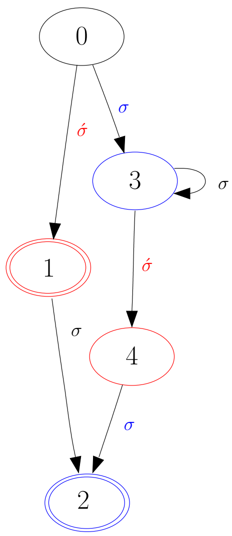

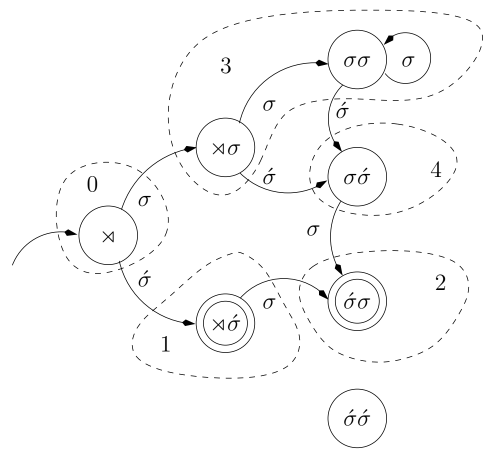

Arabic (Bani-Hassan)

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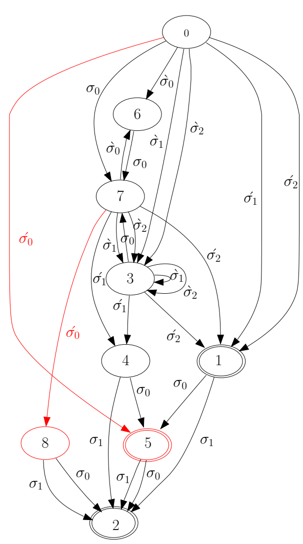

$$\mathcal{G}_{\text{ArabicBH}} = \{\cdots\} - \{\sigma\acute{\sigma}_0\ltimes \mid \sigma \in \sigma_0, \sigma_1, \sigma_2\}$$

$$L_{\text{ArabicBH}} = L_{\{\cdots\}} \cap L_{\overline{\sigma\acute{\sigma}_0\ltimes}}$$

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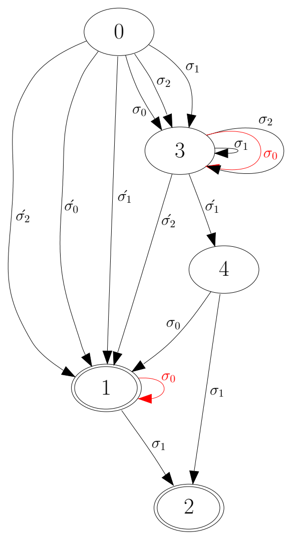
Arabic (Classical)



$\rtimes\sigma_1$	$\overbrace{\sigma_0\cdots\sigma_0}^{k-1}$	$\acute{\sigma_2}\ltimes$
$\rtimes\acute{\sigma_2}$	$\overbrace{\sigma_0\cdots\sigma_0}^{k-1}$	$\sigma_1\ltimes$
$\star\ \rtimes\sigma_1$	$\overbrace{\sigma_0\cdots\sigma_0}^{k-1}$	$\sigma_1\ltimes$

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Strictly Local Stress Patterns

Heinz's Stress Pattern Database (ca. 2007)—109 patterns

9 are SL_2	Abun West, Afrikans, ... Cambodian,... Maranungku
44 are SL_3	Alawa, Arabic (Bani-Hassan),...
24 are SL_4	Arabic (Cairene),...
3 are SL_5	Asheninca, Bhojpuri, Hindi (Fairbanks)
1 is SL_6	Icua Tupi
28 are not SL	Amele, Bhojpuri (Shukla Tiwari), Arabic Classical, Hindi (Keldar), Yidin,...

72% are SL, all $k \leq 6$. 49% are SL_3.

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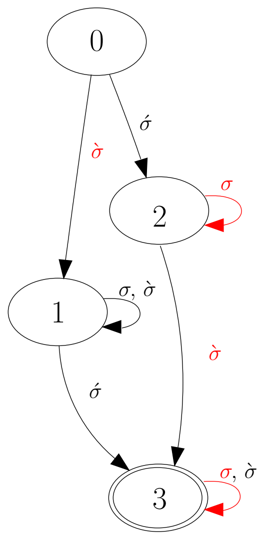
The Problematic Case—Some-$\grave{\sigma}$



$\rtimes\acute{\sigma}$	$\overbrace{\sigma\cdots\sigma}^{k-1}$	$\grave{\sigma}\ltimes$
$\rtimes\acute{\sigma}\grave{\sigma}$	$\overbrace{\sigma\cdots\sigma}^{k-1}$	$\sigma\ltimes$
$\star\,\acute{\sigma}$	$\overbrace{\sigma\cdots\sigma}^{k-1}$	$\sigma\ltimes$

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Locally definable stringsets

$$\begin{array}{ccccc} f \in F_k(\rtimes\cdot\Sigma^*\cdot\ltimes) & & w \models f & \stackrel{\text{def}}{\iff} & f \in F_k(\rtimes\cdot w\cdot\ltimes) \\ \varphi\wedge\psi & & w\models\varphi\wedge\psi & \stackrel{\text{def}}{\iff} & w\models\varphi \text{ and } w\models\psi \\ \neg\varphi & & w\models\neg\varphi & \stackrel{\text{def}}{\iff} & w\not\models\varphi \end{array}$$

Definition 5 (Locally Testable Sets) *A stringset L over Σ is* Locally Testable *iff (by definition) there is some k-expression φ over Σ (for some k) such that L is the set of all strings that satisfy φ:*

$$L = L(\varphi) \stackrel{def}{=} \{w \in \Sigma^* \mid w \models \varphi\}$$

$$\mathrm{SL}_k \equiv \bigwedge_{f_i \notin \mathcal{G}} [\neg f_i] \subsetneq \mathrm{LT}_k$$

Some-$\grave{\sigma}$

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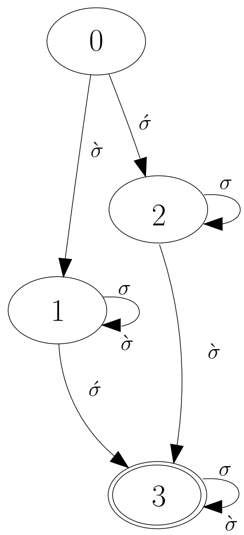

$\varphi_{\text{Some-}\grave{\sigma}} =$

$(\rtimes\acute{\sigma} \vee \acute{\sigma}\ltimes)$	Starts or ends with $\acute{\sigma}$
$\wedge$	
$\grave{\sigma}$	Some $\grave{\sigma}$

LT Automata

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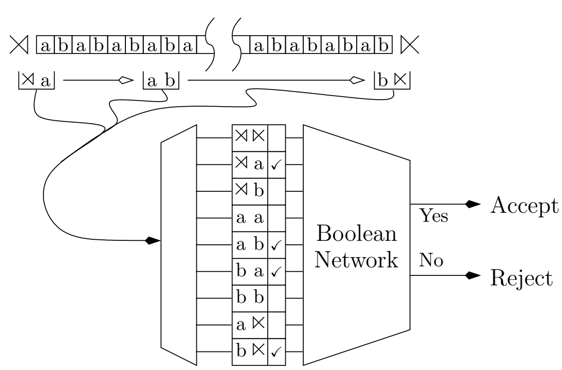

Membership in an LT_k stringset depends only on the set of k-Factors which occur in the string.

Recognizing an LT_k stringset requires only remembering which k-factors occur in the string.

Character of Locally Testable sets

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Theorem 2 (k-Test Invariance) *A stringset L is Locally Testable iff*

> *there is some k such that, for all strings x and y,*
>
> *if $\rtimes \cdot x \cdot \ltimes$ and $\rtimes \cdot y \cdot \ltimes$ have exactly the same set of k-factors*
>
> *then either both x and y are members of L or neither is.*

Definition 6 (k-Local Equivalence)

$$w \equiv_k^L v \stackrel{def}{\iff} F_k(\rtimes w \ltimes) = F_k(\rtimes v \ltimes).$$

LT Hierarchy

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Definition 7 (LT)
*A stringset is k-*Locally Testable *if it is definable with an LT_k-expression.*

A stringset is Locally Testable *(in LT) if it is LT_k for some k.*

Theorem 3 (LT-Hierarchy)

$$LT_2 \subsetneq LT_3 \subsetneq \cdots \subsetneq LT_i \subsetneq LT_{i+1} \subsetneq \cdots \subsetneq LT$$

Cognitive interpretation of LT

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- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) LT_k language must be sensitive, at least, to the *set* of length k contiguous blocks of events that occur in the presentation of the string—both those that do occur and those that do not.
- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the set of length k blocks of events that occurred at any prior point.
- Any cognitive mechanism that is sensitive *only* to the occurrence or non-occurrence of length k contiguous blocks of events in the presentation of a string will be able to recognise *only* LT_k languages.

Arabic (Classical)

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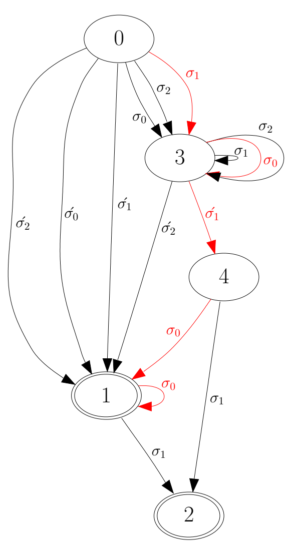

$$\rtimes\sigma_1\overbrace{\sigma_0\cdots\sigma_0}^{k-1}\acute{\sigma}_1\overbrace{\sigma_0\cdots\sigma_0}^{k-1}\ltimes$$

$$\equiv^L_k$$

$$\rtimes\sigma_1\overbrace{\sigma_0\cdots\sigma_0}^{k-1}\acute{\sigma}_1\overbrace{\sigma_0\cdots\sigma_0}^{k-1}\acute{\sigma}_1\overbrace{\sigma_0\cdots\sigma_0}^{k-1}\ltimes$$

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FO(+1)

Models: $\langle \mathcal{D}, \triangleleft, P_\sigma \rangle_{\sigma \in \Sigma}$

First-order Quantification (over positions in the strings)

$$
\begin{array}{lrcl}
x \triangleleft y & w, [x \mapsto i, y \mapsto j] \models x \triangleleft y & \stackrel{\text{def}}{\iff} & j = i + 1 \\
P_\sigma(x) & w, [x \mapsto i] \models P_\sigma(x) & \stackrel{\text{def}}{\iff} & i \in P_\sigma \\
\varphi \wedge \psi & \vdots & & \\
\neg\varphi & \vdots & & \\
(\exists x)[\varphi(x)] & w, s \models (\exists x)[\varphi(x)] & \stackrel{\text{def}}{\iff} & w, s[x \mapsto i] \models \varphi(x) \\
 & & & \text{for some } i \in \mathcal{D}
\end{array}
$$

FO(+1)-Definable Stringsets: $L(\varphi) \stackrel{\text{def}}{=} \{w \mid w \models \varphi\}$.

$$\text{One-}\acute{\sigma} = L((\exists x)[\acute{\sigma}(x) \wedge (\forall y)[\acute{\sigma}(y) \rightarrow x \approx y]\,])$$

Arabic (Classical) is FO(+1)

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Character of the FO(+1) Definable Stringsets

Definition 8 (Locally Threshold Testable) *A set L is* Locally Threshold Testable *(LTT) iff there is some k and t such that, for all $w, v \in \Sigma^*$:*
if for all $f \in F_k(\rtimes \cdot w \cdot \ltimes) \cup F_k(\rtimes \cdot v \cdot \ltimes)$
either $|w|_f = |v|_f$ or both $|w|_f \geq t$ and $|v|_f \geq t$,
then $w \in L \iff v \in L$.

Theorem 4 (Thomas) *A set of strings is First-order definable over $\langle \mathcal{D}, \triangleleft, P_\sigma \rangle_{\sigma \in \Sigma}$ iff it is* Locally Threshold Testable.

Membership in an FO(+1) definable stringset depends only on the multiplicity of the k-factors, up to some fixed finite threshold, which occur in the string.

LTT Automata

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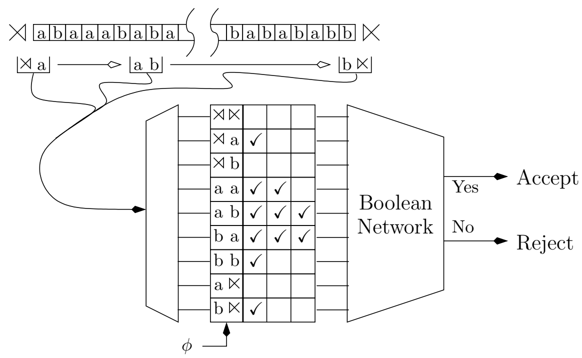

Cognitive interpretation of FO(+1)

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- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) FO(+1) stringset must be sensitive, at least, to the multiplicity of the length k blocks of events, for some fixed k, that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold t.
- If the strings are presented as sequences of events in time, then this corresponds to being able count up to some fixed threshold.
- Any cognitive mechanism that is sensitive *only* to the multiplicity, up to some fixed threshold, (and, in particular, not to the order) of the length k blocks of events in the presentation of a string will be able to recognize *only* FO(+1) stringsets.

No H before $\acute{H}$ is not FO$(+1)$

Primary stress on leftmost heavy syllable

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$$\star\ H \ldots \acute{H}$$

$$\rtimes \overbrace{\grave{L}L \cdots \grave{L}L}^{2kt}\ \acute{H}H\ \overbrace{\grave{L}L \cdots \grave{L}L}^{2kt}\ \grave{H}H\ \overbrace{\grave{L}L \cdots \grave{L}L}^{2kt} \ltimes$$

$$\equiv^{L}_{k,t}$$

$$\star \rtimes \underbrace{\grave{L}L \cdots \grave{L}L}_{2kt}\ \grave{H}H\ \underbrace{\grave{L}L \cdots \grave{L}L}_{2kt}\ \acute{H}H\ \underbrace{\grave{L}L \cdots \grave{L}L}_{2kt} \ltimes$$

First-Order$(<)$ definable stringsets

$\langle \mathcal{D}, \triangleleft^{+}, P_\sigma \rangle_{\sigma \in \Sigma}$

First-order Quantification over positions in the strings

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$$
\begin{array}{ccll}
x \triangleleft^{+} y & w, [x \mapsto i, y \mapsto j] \models x \triangleleft^{+} y & \overset{\text{def}}{\Longleftrightarrow} & i < j \\
P_\sigma(x) & w, [x \mapsto i] \models P_\sigma(x) & \overset{\text{def}}{\Longleftrightarrow} & i \in P_\sigma \\
\varphi \wedge \psi & \vdots & & \\
\neg\varphi & \vdots & & \\
(\exists x)[\varphi(x)] & w, s \models (\exists x)[\varphi(x)] & \overset{\text{def}}{\Longleftrightarrow} & w, s[x \mapsto i] \models \varphi(x) \\
 & & & \quad \text{for some } i \in \mathcal{D}
\end{array}
$$

Star-Free stringsets

Slide 41

Definition 9 (Star-Free Set) *The class of* Star-Free Sets *(SF) is the smallest class of languages satisfying:*

- *Fin* $\subseteq$ *SF.*
- *If* $L_1, L_2 \in SF$ *then:* $L_1 \cdot L_2 \in SF$,
  $L_1 \cup L_2 \in SF$,
  $\overline{L_1} \in SF$.

Theorem 5 (McNauthton and Papert) *A set of strings is First-order definable over* $\langle \mathcal{D}, \triangleleft^{+}, P_\sigma \rangle_{\sigma \in \Sigma}$ *iff it is* Star-Free.

Cognitive interpretation of SF (FO(<))

Slide 42

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) SF language must be sensitive, at least, to both the order and the multiplicity of the length k blocks of events, for some fixed k, that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold t.
- If the strings are presented as sequences of events in time, then this corresponds to being able not only to count events up to some threshold but also to track the sequence in which those events occur.
- Any cognitive mechanism that is sensitive *only* to the order and the multiplicity of the length k blocks of events, for some fixed k, that occur in the presentation of the string, distinguishing multiplicities only up to some fixed threshold t will be able to recognise *only* SF languages.

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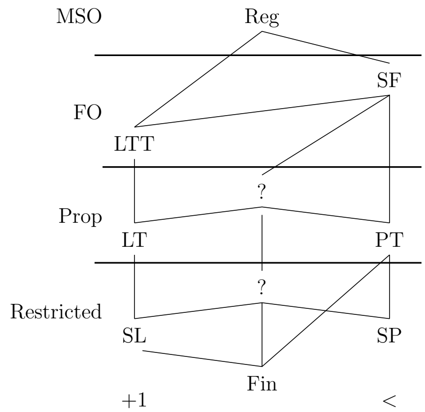
Sub-Regular Hierarchies



Slide 44

Yidin

- Primary stress on the leftmost heavy syllable, else the initial syllable
- Secondary stress iteratively on every second syllable in both directions from primary stress
- No light monosyllables

Explicitly:

- Exactly one $\acute{\sigma}$ (One-$\acute{\sigma}$)
- $\acute{L}$ implies no H (No-H-with-$\acute{L}$)
- σ and $\grave{\sigma}/\acute{\sigma}$ alternate (Alt)
- First H gets primary stress (No-H-before-$\acute{H}$)
- $\acute{L}$ only if initial (Nothing-before-$\acute{L}$)
- No $\acute{L}$ monosyllables (No $\rtimes\acute{L}\ltimes$)

Slide 45

Classifying Conjunctive Constraints

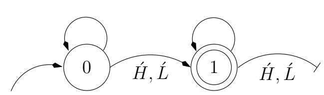
- One-$\acute{\sigma}$ — $(\exists! x)[\acute{\sigma}(x)]$ — ($LTT_{1,2}$)
- No-H-before-$\acute{H}$ — $\neg(\exists x, y)[x \triangleleft^{+} y \wedge H(x) \wedge \acute{H}(y)]$ — (SF)
- No-H-with-$\acute{L}$ — $\neg(H \wedge \acute{L})$ — (LT_1)
- Nothing-before-$\acute{L}$ — $\neg\sigma\acute{L}$ — (SL_2)
- Alt — $\neg\sigma\sigma \wedge \neg\acute{\sigma}\acute{\sigma} \wedge \neg\acute{\sigma}\grave{\sigma} \wedge \neg\grave{\sigma}\acute{\sigma} \wedge \neg\grave{\sigma}\grave{\sigma}$ — (SL_2)
- No $\rtimes\acute{L}\ltimes$ — $\neg\rtimes\acute{L}\ltimes$ — (SL_3)

Yidin is SF

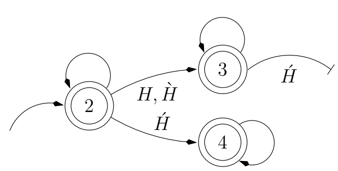
Slide 46

Combining Conjunctive Constraints

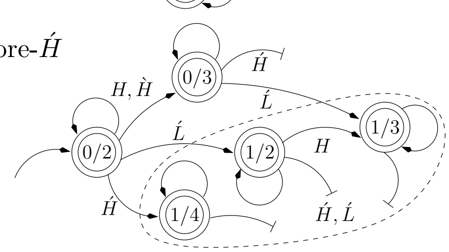
- One-$\acute{\sigma}$



- No-H-before-$\acute{H}$



- One-$\acute{\sigma}$ $\cap$ No-H-before-$\acute{H}$



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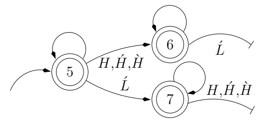
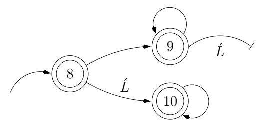
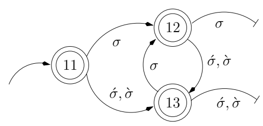
- No-H-with-$\acute{L}$



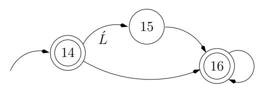
- Nothing-before-$\acute{L}$



- Alt

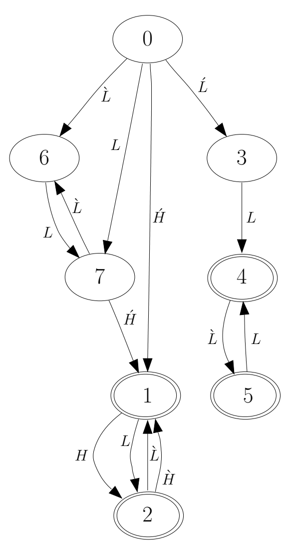


- No $\rtimes\acute{L}\ltimes$



Yidin

Slide 48



Precedence—Subsequences

Definition 10 (Subsequences)

$$v \sqsubseteq w \overset{def}{\iff} v = \sigma_1 \cdots \sigma_n \textit{ and } w \in \Sigma^* \cdot \sigma_1 \cdot \Sigma^* \cdots \Sigma^* \cdot \sigma_n \cdot \Sigma^*$$

$$P_k(w) \overset{def}{=} \{v \in \Sigma^k \mid v \sqsubseteq w\}$$

$$P_{\leq k}(w) \overset{def}{=} \{v \in \Sigma^{\leq k} \mid v \sqsubseteq w\}$$

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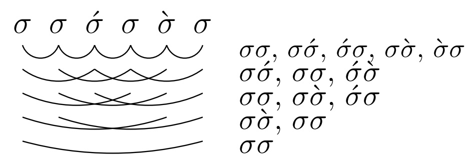

$$P_2(\sigma\sigma\acute{\sigma}\sigma\grave{\sigma}\sigma) = \{\sigma\sigma, \sigma\acute{\sigma}, \sigma\grave{\sigma}, \acute{\sigma}\sigma, \acute{\sigma}\grave{\sigma}, \grave{\sigma}\sigma\}$$

$$P_{\leq 2}(\sigma\sigma\acute{\sigma}\sigma\grave{\sigma}\sigma) = \{\varepsilon, \sigma, \acute{\sigma}, \grave{\sigma}, \sigma\sigma, \sigma\acute{\sigma}, \sigma\grave{\sigma}, \acute{\sigma}\sigma, \acute{\sigma}\grave{\sigma}, \grave{\sigma}\sigma\}$$

Strictly Piecewise Stringsets—SP

Strictly k-Piecewise Definitions

$$\mathcal{G} \subseteq \Sigma^{\leq k}$$

$$w \models \mathcal{G} \overset{\text{def}}{\iff} P_{\leq k}(w) \subseteq P_{\leq k}(\mathcal{G})$$

$$L(\mathcal{G}) \overset{\text{def}}{=} \{w \in \Sigma^* \mid w \models \mathcal{G}\}$$

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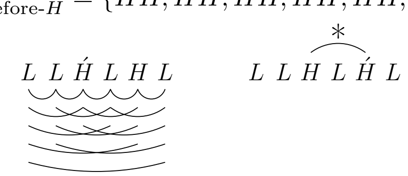
$$\mathcal{G}_{\text{No-}H\text{-before-}\acute{H}} = \{HH, H\grave{H}, \grave{H}H, \grave{H}\grave{H}, \acute{H}H, \acute{H}\grave{H}, \ldots\}$$



Membership in an SP_k stringset depends only on the individual ($\leq k$)-subsequences which do and do not occur in the string.

Character of the Strictly k-Piecewise Sets

Theorem 6 *A stringset L is Strictly k-Piecewise Testable iff it is closed under subsequence:*

$$w\sigma v \in L \Rightarrow wv \in L$$

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Every naturally occurring stress pattern requires Primary Stress
$\Rightarrow$
No naturally occurring stress pattern is SP.

But SP can forbid multiple primary stress: $\neg\acute{\sigma}\acute{\sigma}$

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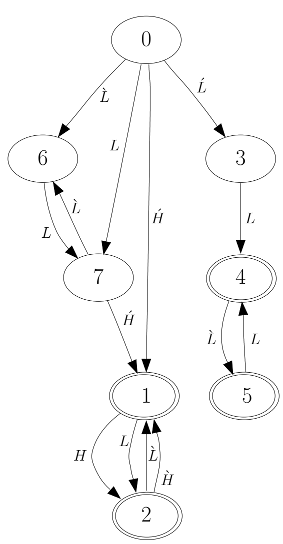
Yidin constraints wrt SP



- One-$\acute{\sigma}$ is not SP
  $\star\ \sigma\sigma\ \sqsubseteq\ \sigma\acute{\sigma}\sigma$
- No-H-before-$\acute{H}$ is SP_2
  $\neg H\acute{H}$
- No-H-with-$\acute{L}$ is SP_2
  $\neg H\acute{L} \wedge \neg\acute{L}H$
- Nothing-before-$\acute{L}$ is SP_2
  $\neg\sigma\acute{L}$
- Alt is not SP
  $\star\ \sigma\sigma\acute{\sigma}\ \sqsubseteq\ \sigma\grave{\sigma}\sigma\acute{\sigma}$
- No $\rtimes\acute{L}\ltimes$ is not SP
  $\star\ \acute{L}\ \sqsubseteq\ \acute{L}L$

Cognitive interpretation of SP

Slide 53

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) SP_k stringset must be sensitive, at least, to the length k (not necessarily consecutive) sequences of events that occur in the presentation of the string.
- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to up to $k-1$ events distributed arbitrarily among the prior events.
- Any cognitive mechanism that is sensitive *only* to the length k sequences of events in the presentation of a string will be able to recognize *only* SP_k stringsets.

k-Piecewise Testable Stringsets

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PT_k-expressions

$$\begin{array}{ccccc} p \in \Sigma^{\leq k} & w \models p & \stackrel{\text{def}}{\iff} & p \sqsubseteq w \\ \varphi \wedge \psi & w \models \varphi \wedge \psi & \stackrel{\text{def}}{\iff} & w \models \varphi \text{ and } w \models \psi \\ \neg\varphi & w \models \neg\varphi & \stackrel{\text{def}}{\iff} & w \not\models \varphi \end{array}$$

k-Piecewise Testable Languages (PT_k):

$$L(\varphi) \stackrel{\text{def}}{=} \{w \in \Sigma^* \mid w \models \varphi\}$$

$$\text{One-}\acute{\sigma} = L(\acute{\sigma} \wedge \neg\acute{\sigma}\acute{\sigma})$$

Membership in an PT_k stringset depends only on the set of ($\leq k$)-subsequences which occur in the string.

SP_k is equivalent to $\bigwedge_{p_i \notin \mathcal{G}}[\neg p_i]$

Slide 55

Character of Piecewise Testable sets

Theorem 7 (k-Subsequence Invariance) *A stringset L is Piecewise Testable iff*

there is some k such that, for all strings x and y,

if x and y have exactly the same set of $(\leq k)$-subsequences

then either both x and y are members of L or neither is.

$$w \equiv^P_k v \overset{\text{def}}{\Longleftrightarrow} P_{\leq k}(w) = P_{\leq k}(v).$$

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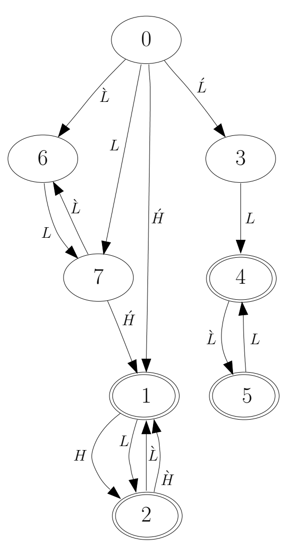
Yidin constraints wrt PT



- One-$\acute{\sigma}$ is PT_2
  $\acute{\sigma} \wedge \neg\acute{\sigma}\acute{\sigma}$
- No-H-before-$\acute{H}$ is SP_2
  $\neg H\acute{H}$
- No-H-with-$\acute{L}$ is SP_2
  $\neg H\acute{L} \wedge \neg\acute{L}H$
- Nothing-before-$\acute{L}$ is SP_2
  $\neg\sigma\acute{L}$
- Alt is not PT
  $\star\ \overbrace{\sigma\grave{\sigma}\cdots\sigma\grave{\sigma}}^{2k} \equiv^{P}_{k} \overbrace{\sigma\grave{\sigma}\cdots\sigma\grave{\sigma}}^{2k}\grave{\sigma}$
- No $\rtimes\acute{L}\ltimes$ is PT_2
  $\acute{L} \rightarrow (\sigma\acute{L} \vee \acute{L}\sigma)$

Cognitive interpretation of PT

Slide 57

- Any cognitive mechanism that can distinguish member strings from non-members of a (properly) PT_k stringset must be sensitive, at least, to the set of length k subsequences of events that occur in the presentation of the string—both those that do occur and those that do not.
- If the strings are presented as sequences of events in time, then this corresponds to being sensitive, at each point in the string, to the set of all length k subsequences of the sequence of prior events.
- Any cognitive mechanism that is sensitive *only* to the set of length k subsequences of events in the presentation of a string will be able to recognize *only* PT_k stringsets.

Yidin wrt Local and Piecewise Constraints

Slide 58

One-$\acute{\sigma}$	$\text{LTT}_{1,2}$	PT_2
Some-$\acute{\sigma}$	LT_1	PT_1
At-Most-One-$\acute{\sigma}$	$\text{LTT}_{1,2}$	SP_2
No-H-before-$\acute{H}$	SF	SP_2
No-H-with-$\acute{L}$	LT_1	SP_2
Nothing-before-$\acute{L}$	SL_2	SP_2
Alt	SL_2	SF
No $\rtimes\acute{L}\ltimes$	SL_3	PT_2

Yidin is co-occurence of SL and PT constraints or of LT and SP constraints

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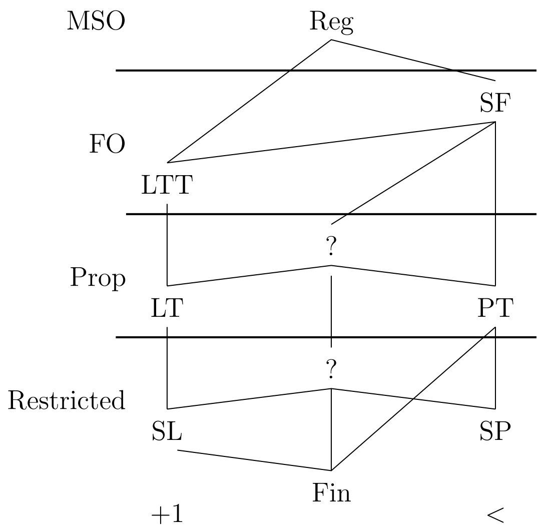
Local and Piecewise Hierarchies



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MSO definable stringsets

$\langle \mathcal{D}, \triangleleft, \triangleleft^+, P_\sigma \rangle_{\sigma \in \Sigma}$

First-order Quantification (positions)

Monadic Second-order Quantification (sets of positions)

$\triangleleft^+$ is MSO-definable from $\triangleleft$.

Character of the MSO-definable sets

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Theorem 8 (Medvedev, Büchi, Elgot) *A set of strings is MSO-definable over* $\langle \mathcal{D}, \triangleleft, \triangleleft^{+}, P_\sigma \rangle_{\sigma \in \Sigma}$ *iff it is regular.*

Theorem 9 (Chomsky Schützenberger) *A set of strings is Regular iff it is a homomorphic image of a Strictly 2-Local set.*

Theorem 10 $MSO = \exists MSO$ *over strings.*

Cognitive interpretation of Finite-state

Slide 62

- Any cognitive mechanism that can distinguish member strings from non-members of a finite-state stringset must be capable of classifying the events in the input into a finite set of abstract categories and are sensitive to the sequence of those categories.
- Subsumes *any* recognition mechanism in which the amount of information inferred or retained is limited by a fixed finite bound.
- Any cognitive mechanism that has a fixed finite bound on the amount of information inferred or retained in processing sequences of events will be able to recognize *only* finite-state stringsets.

Local and Piecewise Hierarchies

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